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UNDERSTANDING ANCIENT DESIGN THROUGH SURVEY: EXAMPLES FROM HADRIAN'S VILLA

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Abstract

The Dipartimento di Architettura of the Alma Mater Studiorum - University of Bologna is carrying out a series of investigations on the Villa Adriana in Tivoli, in collaboration with the Soprintendenza Archeologia del Lazio e Dell'Etruria Meridionale, Italian and foreign Universities and higher education institutions and companies in the field of remote sensing, which have digitally acquired several pavilions of this important UNESCO site since 2013. In this paper we present briefly the first outcomes of numerous documentation campaigns aimed at solving some outstanding problems in relation with investigation lines concerning mixtilinear plan design and complex vaulted spaces. The latter, thanks to a complete digital surveying campaign, can be examined with an high level of accuracy that led to more reliable interpretation of vaulted systems conceived during Hadrianic age.

Keywords

Vaults, Cupolas, Mixtilinear Plan Design, Hadrian's Villa, 3D Laser Scanner

1. *Introduction*

Over the past twelve years, many among Italian and foreign universities and research organizations have carried out laser scanner and photogrammetric surveys on the archaeological remains the Hadrian's Villa complex consist of. Most of the monumental buildings of this Unesco site have been object of accurate surveys, yet still lacking of a more general overview and the necessary datum points, essential to geo-reference the gathered three-dimensional data in an overall framework. It is hoped that this research project could be realized shortly in order to allow a better understanding of the complete area and an in-depth analysis on the development of the ancient settlement.[1]

Nevertheless, in the framework of a more specific research branch – even if devoid of a complete 2D and 3D output of the whole complex[2] – it has been possible to carry out a series of accurate analysis taking the cue from an interesting topic that have always fascinating scholars, to be precise decorated friezes.[3] They were situated in trabeations of some particularly representative pavilions in the Villa that, as is common knowledge, present an original mixtilinear pattern, which inspired numerous formal solution of the Roman and European Baroque,[4] and not only.[5]

In this way, starting from September 2009, we have been able to provide a reinterpretation and further cause for reflection on those elements that are part of the architectural decorations, based on highly reliable and accurate digital

[1] On the topic of the overall evaluation of Hadrian's Villa project based on geometrical analysis, see the following references: Di Tondo (2007), Giuffrida (2007), Caliari (2012), Cinque and Lazzeri (2010), Ytterberg (2013).

[2] For an overview of Hadrian's Villa work-in-progress surveying, see Bertocci (2015).

[3] For a complete bibliography on the topic of decorated friezes, in particular at the Maritime Theatre and Piazza d'Oro, see: Adembri (2013) and Adembri, Di Tondo, Fantini, and Ristori (2014). For geometrical analysis from laser scanner and photogrammetric surveys of the little southern porticoes at Piazza D'Oro and respective friezes, see Adembri, Vidal, and Martínez-Espejo Zaragoza (2012).

[4] On the direct influence of Hadrian's Villa cupolas on Borromini's work, see Portoghesi (1994), in particular at pp. 9-20 and p. 64.

[5] Among scholars and architects that drawn inspiration from the Villa for their projects, a special role is played by John Soane who carried out a detailed survey of the Small Baths complex. See Richardson and Stevens (2000), in particular at pp. 62- 66.

surveys both of friezes and of respective buildings they belong to.

The main buildings, incorporating such innovative solutions based on alternating straight or curved portions of trabeation, or bowed trabeations alternating bent portions and cusps, – recently surveyed in full by the University of Bologna – are the Maritime Theatre (Figure 1), the Piazza d'Oro (Golden Square) and the Serapeum, including the so-called Canopus.

Fig. 1: The Maritime Theatre: an extensive surveying campaign was carried out both on masonries and entablatures, with special regard to friezes

In addition to these, other buildings, whilst presenting the same characteristics in plan based on the alternation between bent or rectilinear elements, are not generally included in the mentioned research branch, namely the pavilion of Venus Cnidia, the Small Baths and Roccabruna.[6]

The main objective of this study is to restore friezes in the corresponding original locations by using digital models fit for virtual anastylosis, especially of curvilinear elements and those parts presenting a peculiar cut of marble with the purpose of assembly and installation and setup (Figure 2).

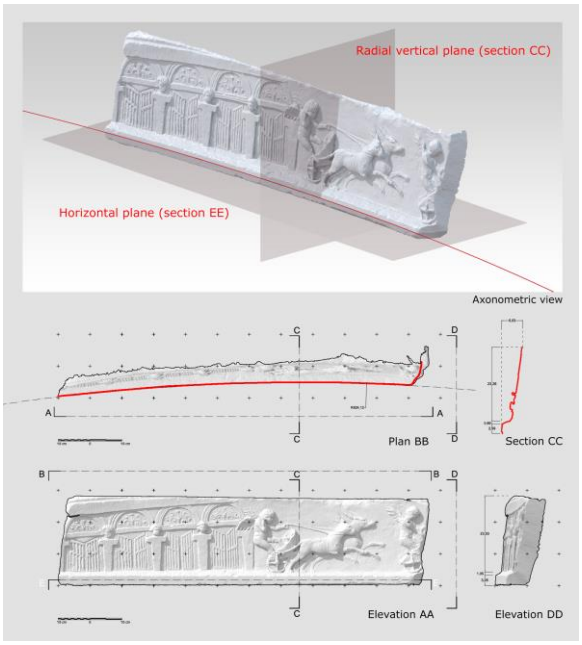

Fig. 2: 3D model of a frieze originally situated on the Maritime Theatre's entablature and now preserved at the British Museum in London (survey by Sergio Di Tondo and Filippo Fantini). The bend radius was extracted by means of reverse modelling applications in order to understand its original position within the mixtilinear plan of Hadrian's Villa

Research experiences carried out until now caused an undeniable logistical and coordination effort, also because of achieving uniformity for data and requiring specific rules for data storage and access, regarding both surveys at architectural scale gathered through terrestrial laser scanner (phase-shift and time-of-flight) and photogrammetric campaigns based on SfM and MVS applications, moreover mesh models from triangulation-based 3D scanners of decorated friezes and other elements being part of entablatures (cornices and architraves). Additionally, depending on the case, there is also graphic material, such as CAD drawings relating to plans, elevations and cross-sections of the

[6] Most of these surveys were carried out during workshops for students enrolled at the Alma Mater Studiorum-University of Bologna, Campus of Ravenna, for first and second cycle degree programmes in *Ingegneria Edile* (Disegno Edile T-2) and *Ingegneria dei processi e dei sistemi edilizi* (Modellazione BIM M), Professors Luca Cipriani and Filippo Fantini. Other students participating to the 3D digital documentation of the Villa are enrolled at the Master in *Progettazione strategica e gestione innovativa per le aree archeologiche*, Accademia Adrianea di Architettura e Archeologia (Professor Pier Federico Caliari).

monument documenting the present state of the site and 3D digital models (mesh or NURBS but also subdivision surfaces and displaced subD)[7] (Figure 3).

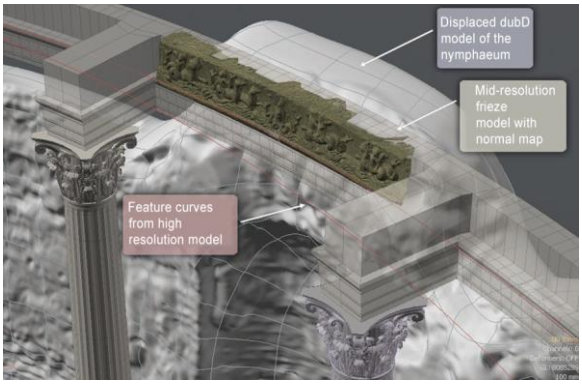

Fig. 3: For a virtual anastylosis of the fragments within a comprehensive 3D model of the masonries, it was necessary the development of optimized LOD models (displaced subD). Detail of the southern nymphaeum at the Piazza d'Oro (Golden Square). Image by Adembri, Di Tondo, and Fantini (2012)

Besides the necessary rigour about the acquisition standards and data processing, we would like to point out a more important critical element emerging since intending to go beyond simple documentation to delve more deeply in the complex field of interpretation starting from reality-based models.

Re-assembly and completion operations or virtual anastylosis entail a new reading and interpretation of the original project, determining the need for scholars to be in the same conditions of the architect whose work in ancient times tried to meet client's requirements who had commissioned the building. Therefore, survey at different scales is useful both for documenting and for cataloguing fragments, besides analysing those crucial elements to reconstruct the ancient project a posteriori (Adembri, Cipriani, Fantini, & Bertacchi, 2015). In order to understand the methodology behind this processing – above all creative but at the same time of a technical nature – it is essential to start interdisciplinary studies with a productive debate and exchange among researchers and experts of vocational education and expertise different from architects and engineers hence complementary to the research.

The line of research presented here is the result of the partnership with the *Soprintendenza per i Beni Archeologici del Lazio*, recently converted into the *Soprintendenza Archeologia del Lazio e dell'Etruria Meridionale*. Over the years this profitable collaboration has led to significant outcomes, examining in depth or contesting the results of previous research, not only through the use of next-generation devices and methodologies, but also and above all thanks to a shared objective to understand the aims and methods characterizing the individual fields of research.

As a logical consequence, this communication constitutes an exchange among scholars towards a sort of common knowledge, not easy to be achieved, which involves the study of their own disciplines in order to deepen topics not completely excelled because of their specific education as architects or archaeologists.[8] This trend in fact endorse the integration of various knowledge in a cultural context, often limited in promoting a new digital aspect for outdated studies.

What has been achieved over the past three years about pavilions characterized by mixtilinear plan is a research path improved over time, also thanks to the contribution of experts and specialists, now moving towards a better understanding of the systems of vaulted and domed spaces made in Roman concrete (*opus caementicium*). This topic is of great historical and architectural interest because it connects and underlines the Hadrian's Villa legacy with history of Western architecture.

As is widely known, according to the Emperor's will, a significant promotion was given to the development of innovative formal solutions based on the extensive use of *opus caementicium*. With such extraordinary building material – made by mixing pozzonalic materials (volcanic rock rich in silicon) with quicklime and water – it was in fact possible to shape innovative and astonishing shapes, breaking the link with tradition not

[7] On methodologies developed to manage interactively high-detailed models, LOD models and models optimized through texture, see Adembri, Di Tondo, and Fantini (2012). On the use of NURBS models for the study of vaulted spaces in the Villa, see Fantini and Pini (2011), Cipriani, Fantini, and Bertacchi (2016).

[8] On the integration of diversified knowledge and the role of philologists within the advancement of knowledge towards an integrated discipline for the understanding of ancient design, see Adembri et al. (2015) and Clini (2012).

appreciating experimental structures made with the "new material."[9]

Previous researches focus on the relationship between marble entablatures, with both decorative and structural function, and concrete domes in the Hadrian period and particularly in the imperial Villa, as demonstrated by several studies (Hansen, 2011; Rakob, 1967; Ueblacker, 1985). Sometimes, as explicitly mentioned by Moneti (1992), long-standing controversy started precisely on the issue whether or not domes made in concrete or of a different nature were present in certain pavilions of the Villa, such as the famous one at Piazza d'Oro, Belvedere dell'Accademia (Accademia Esplanade) and Maritime Theatre.

These analyses, although carried out with rigorous scientific criteria and being still today methodologically emblematic as examples,[10] are based on old surveys not always able to provide the necessary accuracy, in particular as regards elevations and in some cases also concerning the level of detail of decorative elements. This fact causes problems particularly when trying to delve into geometrical and metrological analysis with the aim of finding the compositional master plan and the modular grids the conceptual and constructive process is based on.

Given that some graphic plans were the starting point for many other studies, one can say that incompleteness or mistakes in past surveys (e.g. Rakob carried out only the metric survey of the southern area of Piazza d'Oro, overlooking the rest of the area) led inaccuracy to spread into numerous following studies. That is the reason why some interesting works, carried out with a scientific spirit of inquiry, should be carefully reinterpreted and undergo substantial revisions.[11]

2. *The study of Hadrian's domes: objectives and methodologies*

The presence of numerous vaulted spaces in the Hadrian's Villa (really built or only supposed, as in the case of Piazza d'Oro and the Accademia), along with their intentional multiplicity, has no equal in the ancient world, with the exception of Baiae in Campania, where the emperor passed away.

The Palatine Hill and the Domus Aurea in Rome present elements of major interest from the point of view of an exhaustive and complete reading of the chronological and typological development of the domes in *opus caementicium.*

Also the Pantheon gives interesting hints and keys of interpretation in relation to the experimental shapes present at Hadrian's Villa: the traditional studies on the subject have been proved wrong and, contrary to what stated by previous specialist literature, new exams on brick stamps proved that the project and the construction of the lower part of the rotunda date back to the late reign of Trajan.

The first ancient Roman brick stamp goes back to the age of Hadrian and is located at the base of the dome (Heinzelmann, 2009): for this reason this should be taken into account in the reappraisal of Hadrian's own contribution to the renowned monument, advocating instead the architect Apollodorus of Damascus. It's well known from sources that he was involved in a heated dispute against the emperor specifically in relation to the design of the domes (Cipriani, Fantini, & Bertacchi, 2013).

The Pantheon was concluded in 125 A.D.; the construction of Hadrian's Villa is traditionally divided into two main phases, and a third one for the conclusion of the works, summarized as follows:[12]

- 118 A.D./121 A.D.: Maritime Theatre, Hospitalia (Hall of the Cubicles), Poecile (East-West Terrace), Cento Cemerelle (Hundred Chambers), Eliocaminus Baths, Imperial Palace, Small Baths as well as Great Baths, the two Libraries (Latin and Greek) and the Stadium (Garden Stadium).
- 125 A.D./128 A.D.: Piazza d'Oro, Temple of Venus with Nymphaeum, Stadium

[9] On the innovative nature and application in civil sector of the vaulted structures in Roman concrete, particularly with reference to the Domus Aurea and the Domus Augustana, see Ward-Perkins (1974), in particular at pp. 59-64 of the 2008 reprint.

[10] It is difficult to find further in-depth studies on references, iconography, and functional or constructive interpretation of spaces and decorative finds.

[11] Some assumptions by Jacobson (1986) should be reconsidered since he based his geometrical analysis on dated surveys.

[12] This three chronological phases are according to Calandra (2013), but the topic is still debated among scholars. For different points of view, see Gizzi (2002), Hoffmann (2009), Di Tondo (2007) and Caliari (2012).

(completion), Canopus-Serapum, Winter Palace, Tempe's Pavillion, Doric Pillars, Roccabruna, Accademia, Odeon, Greek Theatre and the Palestra.

- 133/134 A.D.: Various works for completion; and the Egyptian-like area aligned with the Vestibule (the so-called Antinoeion).

In the light of the above, particular interest is raised from a reinterpretation of the development of experimental and innovative shapes of domes and vaults built in the Villa, in relation with one of the most impressive construction site of the Roman world, namely the conclusion of the dome of the Pantheon.

Whatever the reasons inciting to perform a hectic implementation at the Villa (Adembri, Cipriani, & Fantini, 2016), a further aim of this study deals with the in-depth analysis of the design evolution of the complex. In other words, it is possible to state that the late chronological development of the various buildings (approximately built from North to South) corresponds to an increasing in experimentation and to the major monumental nature of the building (Figures 4 and 5), for example in the second period, in particular in the vestibule at Piazza d'Oro (Figure 6) and probably in the dome of the southern area of the complex, in the great dome of the Serapeum and in the Belvedere dell'Accademia.

In addition to these general considerations, it is then necessary to point out another purpose of latest studies, that is to interpret vaulted space by using reality-based models, useful to extract essential information on building techniques and design methods with *opus caementicum*.

In this sense, the digital survey technologies and 3D models help performing a complete documentation, otherwise very difficult through traditional methods. First of all it involves the correct acquisition of information about construction phases and actual execution of the domes,[13] but also provides accurate data concerning metric dimensions, in order to deepen the geometrical genesis behind the structures.

Did the same geometric pattern exist in different pavilions? Can a common construction technology be detected in various buildings in the Villa? Or else, is it possible to imagine that within a common master plan coexisted different schools of thoughts, peculiar for each working group, operating with creative independence?

There is another important issue that concerns the direct literary sources on the construction of Roman domes. In fact, in addition to in-depth knowledge of the shape of the remains thanks to modern remote sensing technologies, one should consider analysing some written sources on the design of vaulted spaces survived to this day.

Giangiacomo Martines (2014) draws attention to the fact that the construction of thin domes in Roman concrete during the age of Nero, was the consequence of a series of progresses not only in the field of material technology, but also and above all in relation to the design methodologies.

The treatise on vaults by Hero of Alexandria is the only ancient source specifically dedicated to the topic of which scholars have gained knowledge.

Unfortunately it has been irretrievably lost; according to the historians, the work would predates not long before the Hadrian's principate (second half of the first century A.D.).

This treatise, cited in the work entitled "*On the Sphere and Cylinder*" by Archimedes, explicitly refers to geometric shapes which are very unusual compared to the formal collection of the Classical age, for instance the parabola:

"*And one can draw the parabola using the compass invented by our master, (the mechanical engineer) Isidore of Miletus, having all been tested by him in the commentary that he wrote on Heron's treatise on domes*" (Martines, 2014, p. 280).

At this stage, it seems quite certain that Nero's architects Severus and Celer, in charge of the construction of the famous octagonal hall of the Domus Aurea, possessed knowledge learned through reading the text by the Alexandrian mathematician, who – according to Martines (2014, p. 288) – also provided in the text a practical component relatively to the construction of the ribs, the formworks and the carpentry in general, besides concepts about geometry and statics applied to solid geometries and three-dimensional figures.

As examined in previous studies (Bianchini & Fantini, 2015), it seems that, even in the midst of the cultural climate of the Hadrian age and particularly at the Villa, there were architects capable of applying Heron's method of calculation.

[13] Some previous studies on Hadrianic vaults and cupolas, such as Cairoli Giuliani (1975), are still substantial for the interpretation of technological aspects.

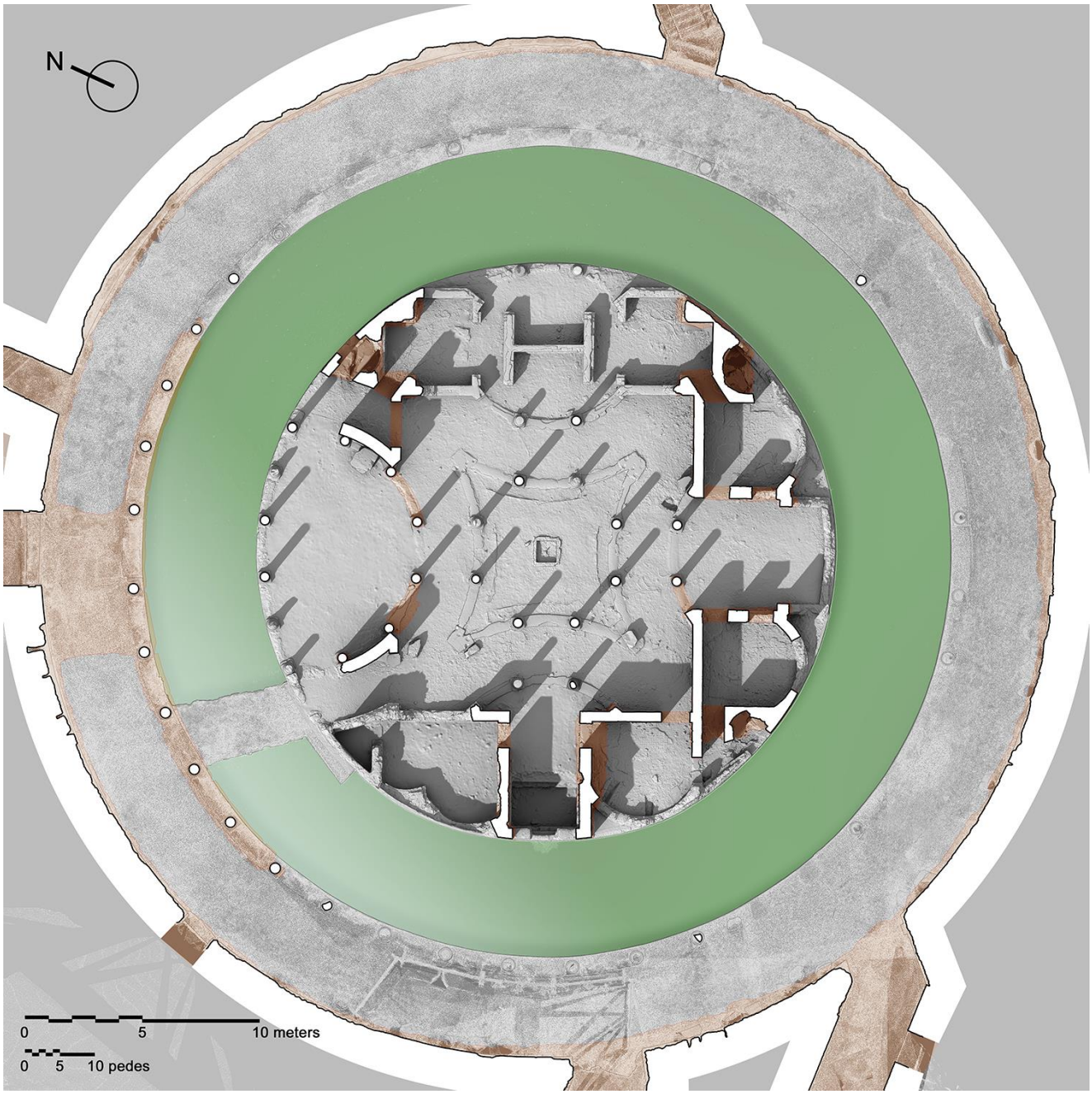

Fig. 4: The Maritime Theatre is characterized by an original mixtilinear shape with an alternation of straight and bended sides: this feature is particularly evident in the portico. This building was built during the first constructive phase of the Villa and represents a first "embryonic" experiment by Hadrian's architects, then followed by further development involving also the elevations (survey by Sergio Di Tondo, Luca Cipriani and Filippo Fantini; mesh modelling by Vincenza Carollo; drawing by Gianna Bertacchi and Silvia Bertacchi)

Therefore, one of this research objectives is to understand whether the remarkable shapes of Hadrian domes can be the result of the development of what conceived by Severus and Celer on the Oppian Hill.

Obviously, with the aim of verifying the reliability of this hypothesis it is fundamental to study a series of clues based on a possible formal and geometrical background in common with buildings with octagonal plan (regular or mixtilinear), in the absence of written sources dealing with that topic. To this purpose, it is interesting to note that before Hadrian's dominion, during Nero's time, thermal baths were the most formally original public buildings.

An example could be the so-called Baths of Nero in Pisa, where the small octagonal *sudatio* (hot room) presents a dome with nine extended

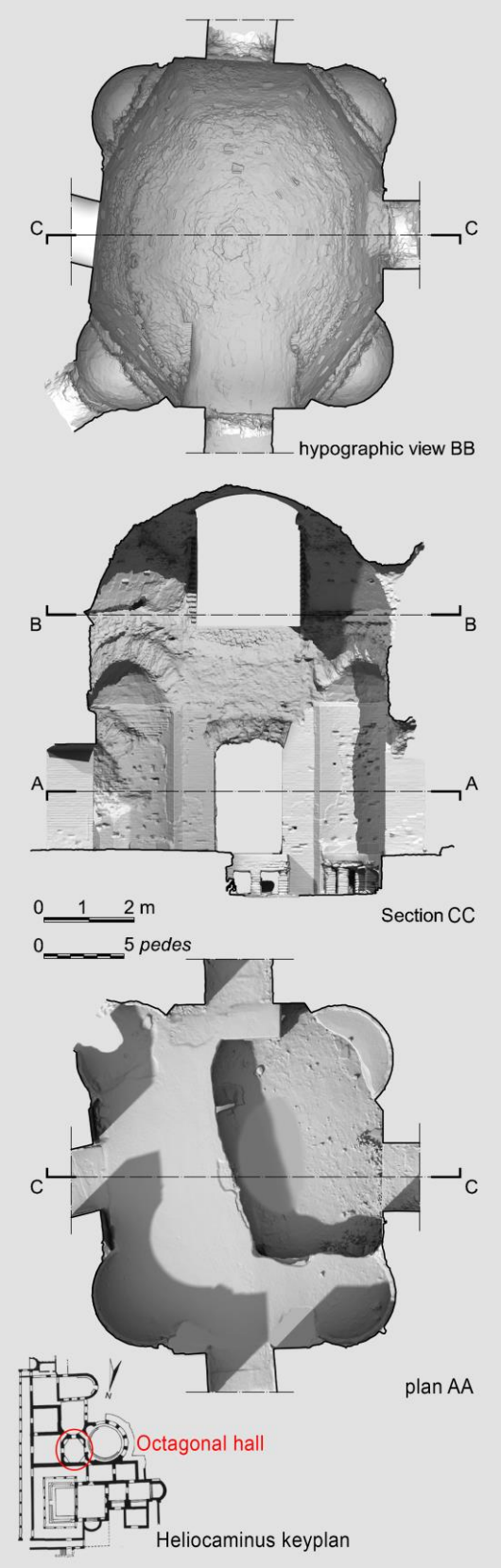

Fig. 5: The so-called Heliocaminus Baths belonged to the first nucleus of the Villa. The main distribution hall (an irregular octagon with four niches) presents some elements that will become more complex in the second constructive phase of the Villa

Fig. 6: The Entrance Vestibule of the Piazza d'Oro is widely considered the first known example of ribbed vault. Its plan is octagonal with an alternation of semi-circular and rectangular niches. It represents a more complex and refined solution rather than former examples at the Villa

Fig. 7: Nero's Baths in Pisa. Detail of the cloister vault intrados with eight windows and a central oculus on the top. Photo by S. Bertacchi

holes, included one central oculus on the top and eight rectangular windows, one in each slice of the domical vault (Campus, 2015) (Figure 7).

A further subject on which it is important to pay attention is the analysis of the formal shape and constructive genesis of Hadrian vaulted spaces; this is the matter of a long-standing debate and some scholars, for example Viscogliosi (2006, p. 332), expressed their doubt about the presence of meticulous geometric patterns behind the conceptual and constructive process of the most daring experiments, such as in the case of the intrados of the Octagonal Hall at the Small Baths: "*It is impossible to describe the domes at Hadrian's Villa from a geometrical point of view, and in particular the so-called Apodyterium of the Tiburtine Small Baths*."

Similar opinion is shared by Ward-Perkins (1974, pp. 85-86) who thinks that "*the architect could take liberties in detail solutions except for elevations and vertical dimensions. Examples such as the vaulted spaces in the Small Baths and Piazza d'Oro seem to prove otherwise, that is a high awareness for the design of the plan and elevations*" (Figure 8).

The research carried out to date, in addition to the general aim of a reliable and extensive 3D documentation of each pavilion or building complex, has the further purpose to provide new interpretations on morphological and dimensional aspects, as well as on the development of new typologies as happens in the vestibule of Piazza d'Oro or the half dome of the Serapeum, first examples of ribbed vaults, then used in various other ancient buildings and very common during Renaissance[14] and Baroque (Portoghesi 1994).

3. *Investigation tool: reverse designing*

The study on Hadrianic cupolas is the product of an integrated methodology based on two reading keys: the first can be defined as "cultural", the second as "technical".

The first aspect is based on the achievement of complete and exhaustive 3D representations, seen as a sort of data storage in which several facts are encrypted. Every building is the result of different conceptual phases (namely a design method, typical of a specific age, applied by the architect) used to solve heterogeneous issues. In many recent studies, the matter concerning architects' tasks – which begins when taking specific requirements into consideration (functional, aesthetic, economical, etc.) – is considered less important in comparison to the achievement of a complete and pure documentation aimed at improving the knowledge concerning different modifications occurred during the lifetime of a building, namely restorations, anthropic alterations and damages, deterioration phenomena.

On the contrary, before deepening these issues, it is relevant to underline the importance of reaching of a wider knowledge whose starting point consists on the understanding of how the architect solved – through applied geometry, calculations and architectural typology – problems concerning the requirements of a building.

The second aspect deals with a correct and complete usage of reverse modelling (RM) applications as instruments to investigate and validate hypothesis on graphic schemes responsible of the final supposed shape of a building.

The investigation on Hadrianic vaults is therefore a perfect investigation field for testing this approach where 3D models from sensors and RM applications may facilitate geometric analysis aimed at understanding the "algorithm" that ruled both plan and elevations (the presence of a specific ratio linking plan and section was already mentioned for the Trajan-Hadrianic Pantheon), as well as the presence of modular grid used to control the dimensioning of the building.

In accordance with Vitruvius (Vitruvio, 1st BC/2002, I, II, 1-9), the first step of the investigation is to split the built object into different design steps, each one characterized by a global coherence that started from the achievement of the plan shape and then elevation/section: therefore using reverse modelling applications for the realisation of three kinds of graphic schemes or constructions, which in ancient times related to the three aspects of the so called *Dispositio*.

This term is used by Vitruvius to describe three features of the designing process: *Ichnographia*, *Orthographia* and *Scaenographia*. They all correspond to different steps of designing, drawing and planning of the building.[15]

[14] On the development of new typologies of domed buildings with octagonal plan (cloister vaults or ribbed vault), see De Angelis D'Ossat (1936).

[15] It is safe to say that *Dispositio* was a sequence of steps or tools, linked to one another and intended at first to define the plan, followed by the façade and lastly modifying and displacing the elements so as to fix them according to optic laws. Hence, the three aspects were actually a set of complex and well structured, graphic constructions, rather than just the drawing of a plan, or a façade of a specific building. For an exhaustive and updated interpretation of the design process, see Adembri et al. (2015).

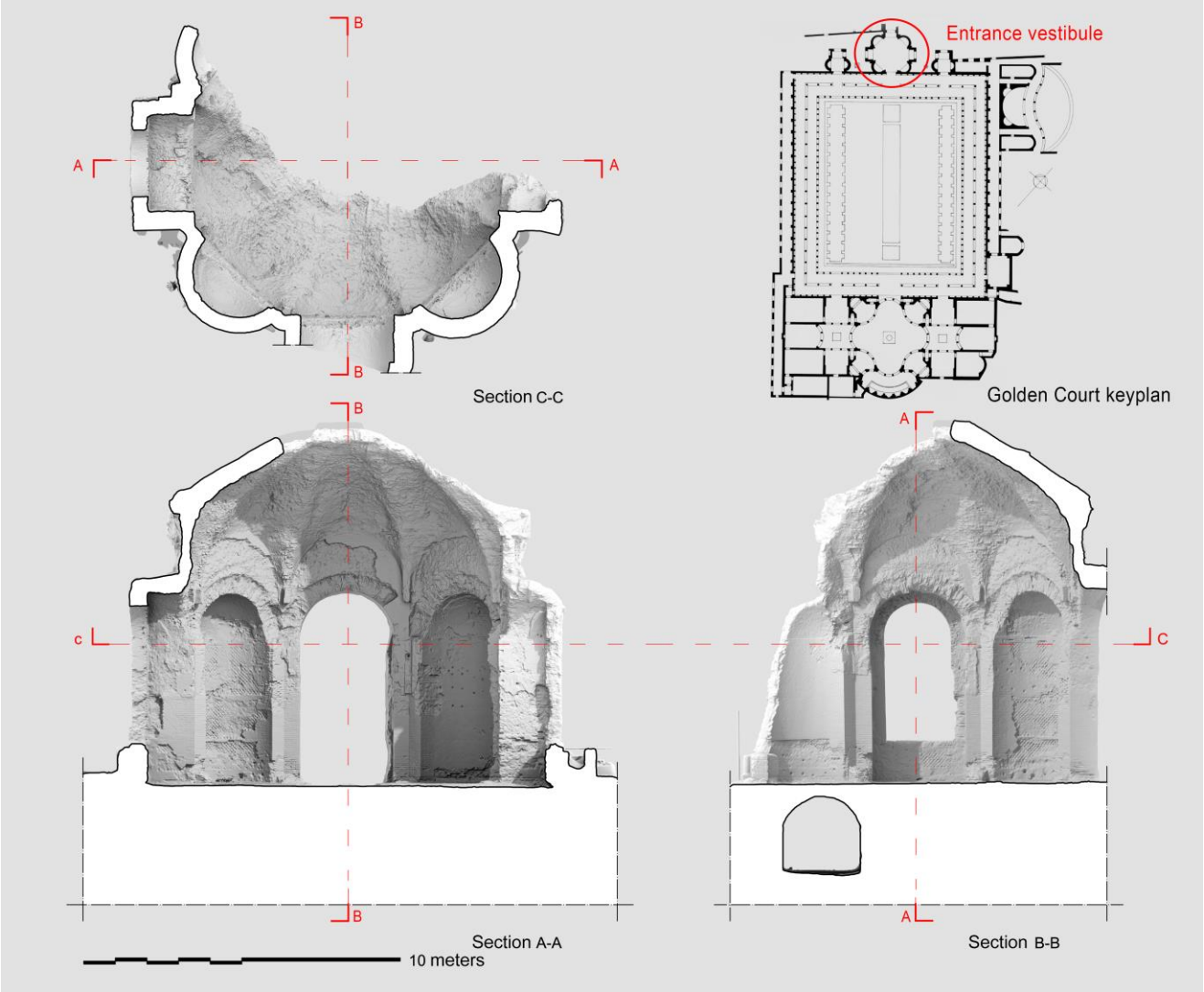

Fig. 8: 3D model of the Entrance Vestibule at the Piazza d'Oro (survey by Sergio Di Tondo, Luca Cipriani and Filippo Fantini; point cloud processing by Luca Grossi; mesh modelling by Simone Vianello)

In this sense when having the 3D mesh of a specific building or a part of it, the way cutting planes are set into the digital environment is crucial, both horizontally and vertically: they have to produce 2D polylines or splines after which a reverse designing process starts.

Just a few numbers of the full set of graphic constructions –we are sure ancient architects applied to achieve a suitable architectural design – survived since they are described in *De Architectura* by Vitruvius and in a very limited number of other written sources.

They deliver the importance of the application of a methodology, based on an in-depth Euclidean knowledge, for the achievement of a proper, easy to replicate and smart shape. In the case of the Small Bath's Octagonal Hall (Figure 9), as well as Piazza d'Oro's vestibule are the product of a refined knowledge of ancient geometry: inscribed and circumscribed polygons to internal and external circles, as well as a progressive blending of shapes from the drum up to supposed spherical caps reflect the architect's skill in applied geometry.

Reference planes and cut planes are not to be used just for the achievement of a correct, but impersonal, set of drawing and sections: these planes have to correspond to the designed sections (not just drawn) by the architect and possibly should be the base for an heuristic phase in which the researcher tries to understand in which way ancient architects applied their knowledge to solve practical problems.

Two other aspects have to be taken into account during the investigation process based on the drawings extracted from highly detailed mesh

models from active or passive sensors: they deal with other two concepts used by Vitruvius that suggest the idea of modularity and proportion. They are *symmetria* and *eurythmia*: the first term refers to a quality that can be described as a general metric coherence of the building, achieved over a grid-based design (Pinho & Xavier, 2013, pp. 83-103).

Fig. 9: Small Baths complex, external view: the Octagonal Hall (on the right) as well as the rest of complex have been surveyed both internally and from outside with special attention to extrados surface through the photogrammetric device 3D Eye (photos by Jorge Martinez-Piqueras and Filippo Fantini)

Eurythmia is a second quality of the construction: it concerns proportions to provide the building with a harmonic composition among the building's main elements and the single parts.

Symmetria is achieved through the application of a grid, made of *posótēs*, but *eurythmia* for is responsible of the presence of a global and local structure of proportions (Figure 10).

Proportions are achieved through a modular grid that eases the composition process and the calculations needed for the distribution of all the areas and measurements that will form to the final construction, forming part of its aesthetic and fulfilling specific requirements.

RM applications provide a robust set of tools designed for tracing graphic elements in particular circles (but also lines, planes, vectors and points) starting from a highly detailed mesh model even if the structures are ruined and altered; in the case of the Small Baths as well as other central plan constructions it is sensible to compare the general measurements of the plan with the elevation, in order to detect if the radius of a best-fitting circle obtained "slicing" a dome matches other relevant measures contained in the plan. Through such a comparison it is then possible to achieve what Duncan-Jones calls "*conjectured target feet*" (Duncan-Jones, 1980, p. 129): when these measurements – once converted into Roman units – correspond approximately to round numbers of *pedes*, this represents relevant hints for the completion of the missing parts of a building.

Among the researches focusing on the mixtilinear halls of Hadrian's Villa and the implication of grid-based design for the development of the ichnographia, Rakob (1967) and Jacobson (1980) provide interesting suggestions, that cannot be totally confirmed by the measurements extracted from the highly detailed digital model. For Rakob the basic module is equal to 5 *pedes* and it was used by the ancient architect for the construction of the octagonal shape starting from a square whose edges are 35 *pedes* and diagonal of approximately 50 *pedes*. Jacobson (1980, p. 80) uses an array of 9 circles inscribed inside a grid formed by squares of 32 *pedes* (Figure 11). A different and more fitting measure of the curved sides is proposed in this paper, achieved through the calculation of an average based on best-fitting circles that is equal to 13.5 *pedes* (see Figure 10). It means that through the synergy of active sensors and RM applications it is possible to improve the accuracy of the geometric interpretation,[16] since mentioned scholars have supplied the reader with measures respectively diverging 73.9 cm and 44.34 cm from our last survey. Taking into consideration that the distance between two opposite lunettes of the hall is about 10.34 m, it is evident the magnitude of the interpretation mistakes that can affect such geometric analysis, converting them in pure geometric exercises, very far from being useful to restorers, art historian, etc.

Moving the investigation towards the *orthographia* of the octagonal hall it is possible to notice other interesting aspect of its design: the two circumferences obtained through best-fitting circles drawn along the vertical planes passing through the midpoints of the bended sides of the octagon are approximately equal 35 *pedes*: it is equal to the side of the fundamental square on which the plan is based (Figure 12 a).

[16] For a recent research about the geometrical interpretation of vaulted spaces through integrated methodology, see Bevilacqua, Caroti, Martínez-Espejo Zaragoza, and Piemonte (2016).

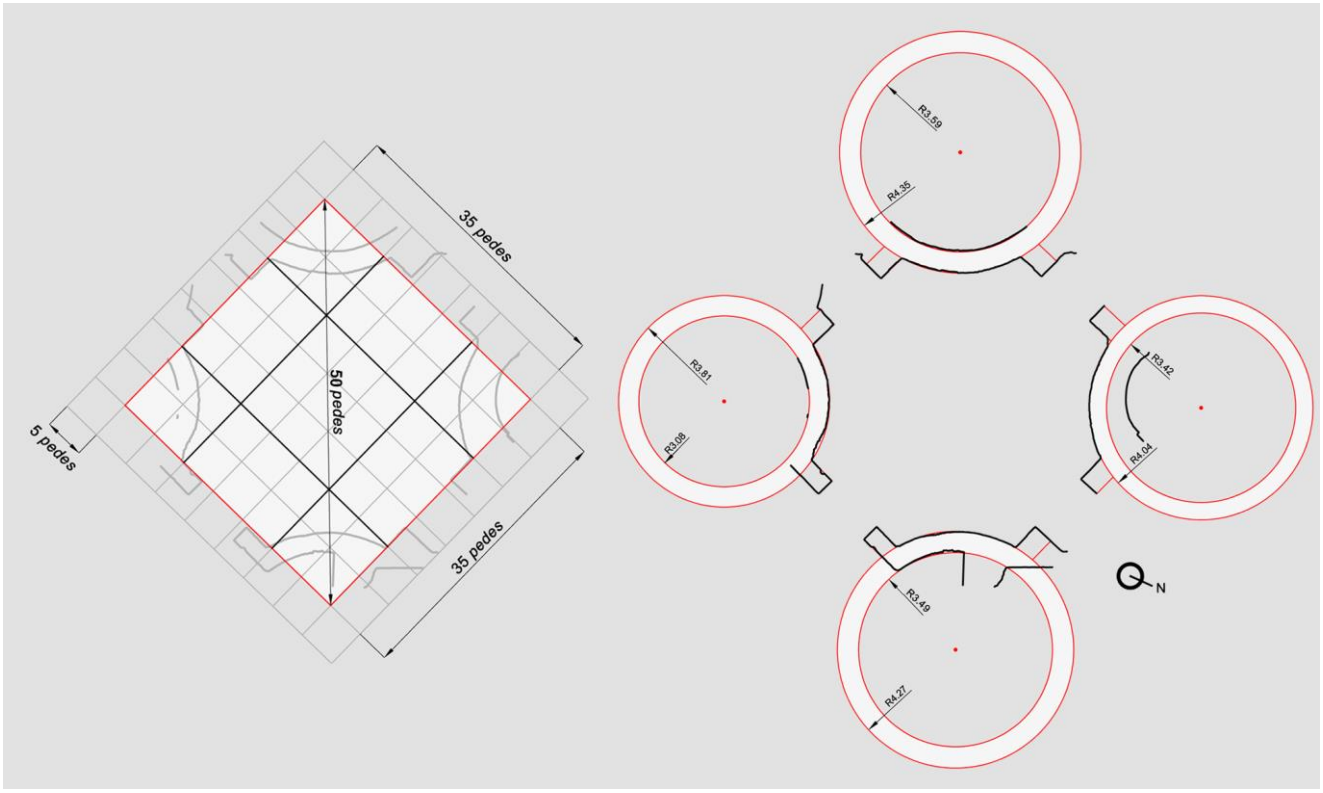

Fig. 10: Interpretation schemes: on the left, the modular grid in accord to the analysis by Rakob (1967); on the right, the best fitting circles extracted through a reverse modelling application

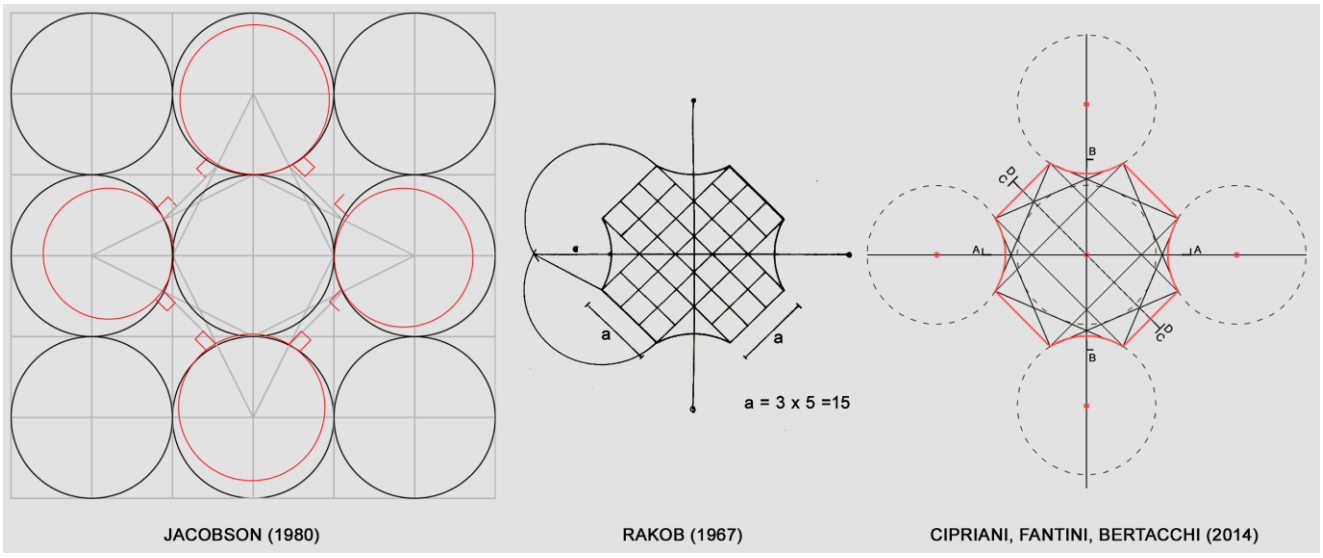

Fig. 11: A comparison between the geometric analysis of the Octagonal Hall's plan: some of the previous interpretation schemes were affected by severe errors due to lack of accuracy during surveying activities

So it seems that the architects may have used a similar geometric pattern for both plan and elevation. It is very difficult, due to the state of conservation of the masonries, the achievement of an exact result, but in any case, from the set of contour lines extracted from the mesh it is possible to understand that the drum mixtilinear shape is converted into a series of linear elements once become a dome, with the exception of the restored parts (Figure 12 b).

In the axonometric geometric scheme (Figure 13 a) it is shown a possible interpretation of the area where bended sides of the plan turn into a side of the vault: this area can be interpreted as the resulting curve of the intersection between the cylindrical surface extruded from the plan and

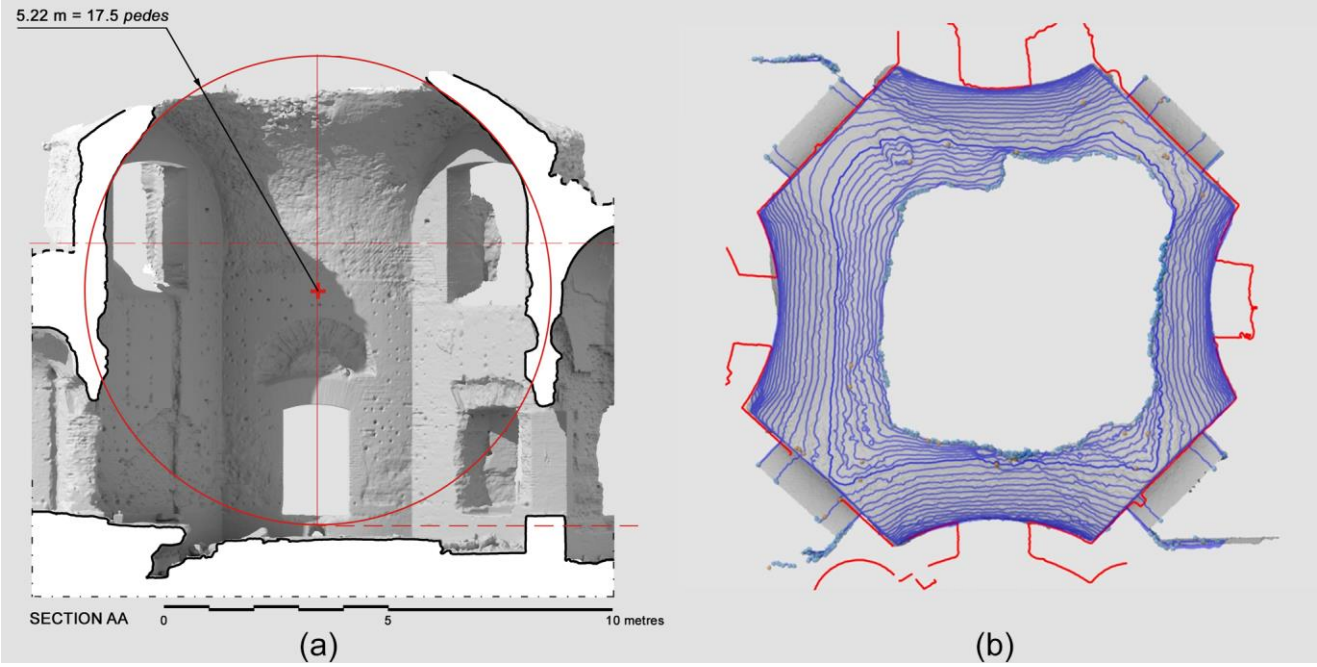

Fig. 12: (a) best-fitting circle passing through vertical section: the diameter is equal to 35 *pedes*. (b) Hypographic view of the Octagonal Hall's dome: the contour lines highlight the presence of not-homogenous irregular shapes due to restorations (upper part of the image). The original shape, even if not clear due to the deterioration phenomena, is preserved in the area on the bottom of the image, where contour lines are more linear

a cylinder with a 35-*pedes* diameter whose axis is parallel to the ground. The double curvature curve c (twisted curve) is then intersected with the impost plane in point A and B. In Figure 13 b, a first surface – forming the collapsed dome – is defined: this patch is bounded by three curves t, and the part of twisted curve c between point A and B. Curve t and u are obtained by intersecting the cylinder with horizontal axis with two vertical planes α and β. Their orientation is far from being clear but to a closer look to the contour lines in Figure 12 b it is safe to say that they could pass respectively through A and B and a vertex of a dodecagon or another regular polygon following the example of the Domus Aurea (Lancaster, 2005, p. 43). In accord to the general flow of the dome provided by the reliable part of the contour lines (Figure 12 b), it seems that at the height of the upper part of the lunettes the mixtilinear octagon turns into the square EFGH (Figure 13 c); the intersections between the side EH and the curves t and u (points C and D) form a bended profile that could be easily connected with the curve v of the lunette.

The last step of this hypothesis consists in the connection between the square EFGH and the dodecagon b (Figure 13 d).

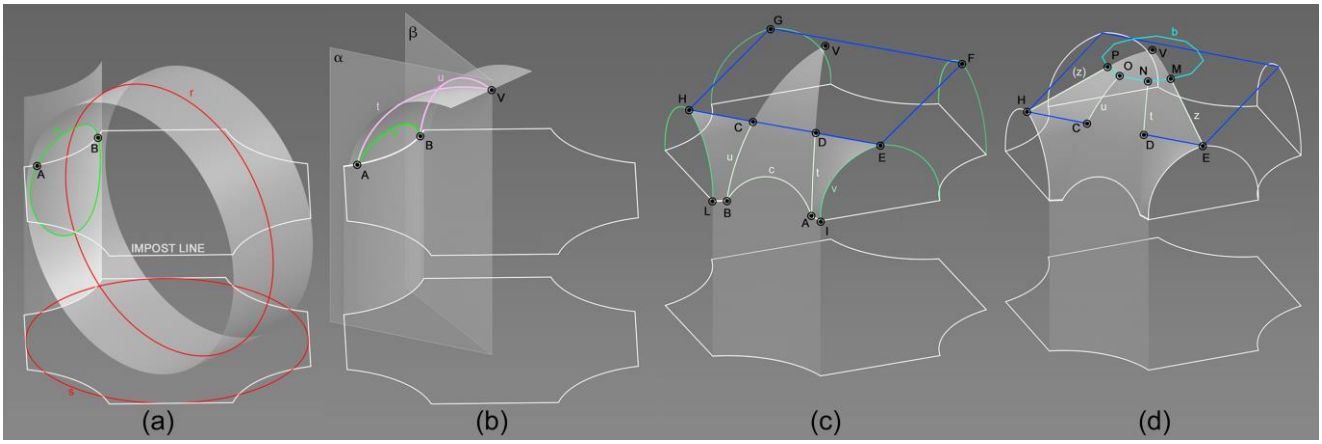

Fig. 13: Hypothesis on the geometric sequence used by the ancient architect for the construction of the cupola

4. Conclusions

These assumptions provide a general framework for the achievement of a synthetic interpretation of the huge amounts of data available through reality-based models of complex shapes. In this sense it is important to establish a coherent strategy for the study of such a complex domes, since many practical problems emerged at the moment of using such dense and accurate meshes from laser scanner and photogrammetric applications based on SfM-MVS solutions.

The greater the amount of data, the greater the problems are to solve and the longer it takes to manage them; for this reason it is important to reach common standards and terminology for data interpretation. In this general framework, RM applications play an important role even if they were developed in different professional fields and with different purposes. Such tools, if complemented with what we called "cultural reading key", perfectly match the objectives of data interpretation paying particular care to the three steps summarized below:

I. Correct definition of reference items (planes, vectors and points) necessary for the extraction of sections in accord to the original ancient design. Starting from a correct interpretation of the plan (conjuctured target *pedes*, *posótēs* and geometric constructions) it is possible to find similarities in the elevation design (*orthographia*);
II. The re-drawing operation of specific elements of a building starts from the automatic detection of specific geometric features, but in many cases (in particular with *opus caementicium* roofings) the deterioration made impossible a direct use of such tools. For this reason it is needed a sort of "hint" from ancient treatises on geometry and mathematics since they included several strategies aimed at solving recursive problems concerning architecture and constructions more in general (machines, weapons, etc.);
III. A final tool is then needed in order to compare idealised models and shapes (based on NURBS, solids, subdivision surfaces, etc.) with the corresponding original mesh, delivering higher accuracy and reliability to the whole reconstruction and interpretation process (Figure 14).

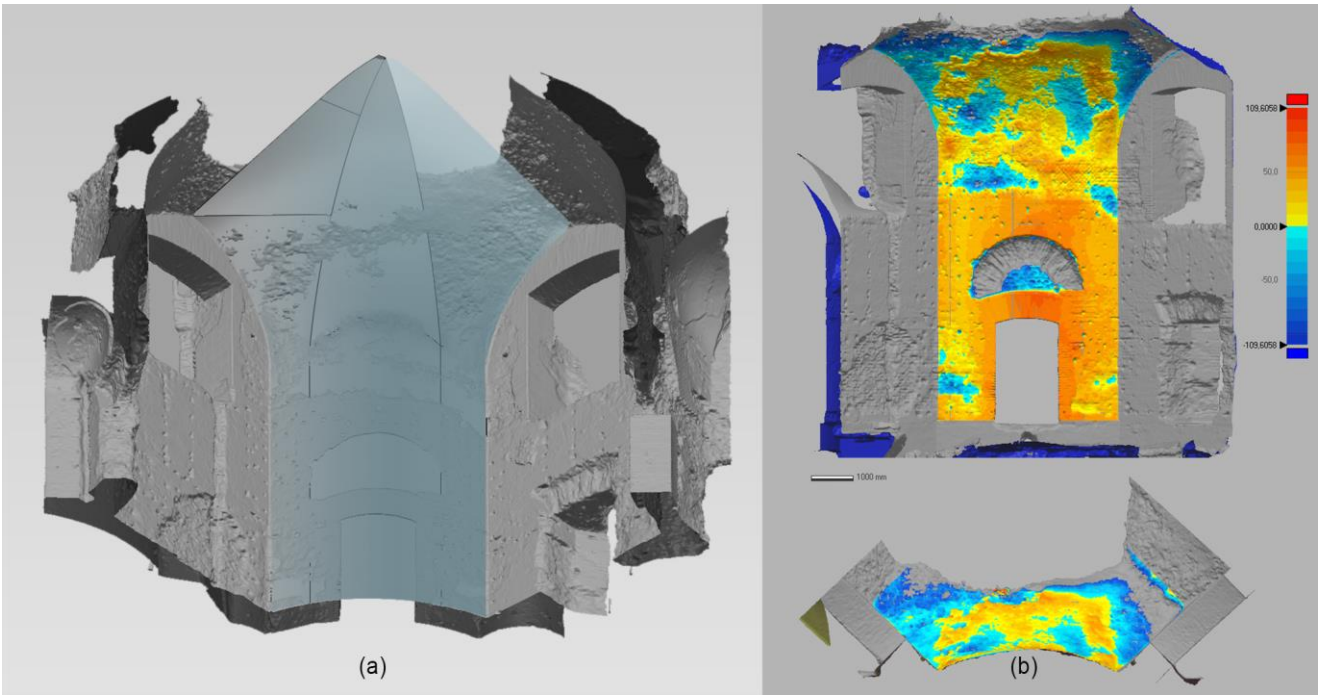

Fig. 14: (a) Overlapping models: in grey colour, the high definition mesh and in azure the NURBS model. (b) Deviation between the original mesh of the more preserved sector of the dome and the NURBS idealized model obtained with reverse modelling procedures

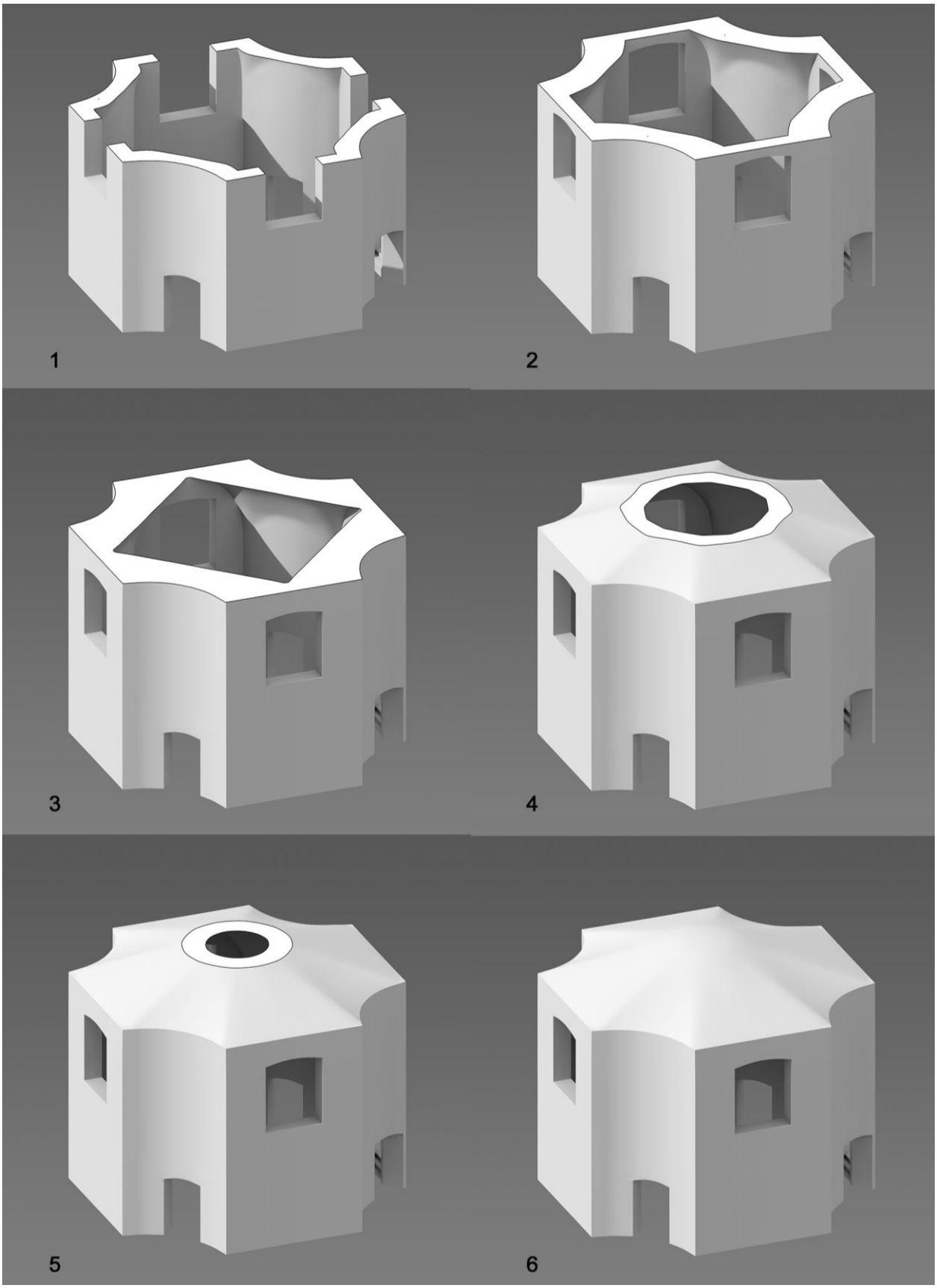

Fig. 15: Summary of the main constructive steps of the Octagonal Hall at the Small Baths

In the case of the Octagonal Hall the second point is crucial since contour lines facilitate the understanding of how ancient architects obtained the perception of the geometric continuity of the vault even if it was no more than a skilful fake (*scaenographia*).The reason is due to the blending from a convex wall to a the concave spherical cap on the top of the dome, achieved through the use of simple ruled surfaces, that approximately converge on an irregular dodecagon on the top (Figure 15). Without a complete laser scanner or photogrammetric survey this aspect is not clear at a naked eye.

In the frame of this vast architectural renovation process carried out by Hadrian, the octagonal hall of the Small Baths in Tivoli plays a fundamental role since it can be considered somehow as halfway between the very first example provided by the Domus Aurea's Octagonal Hall and other later examples, i.e. the so-called Temple of Minerva Medica in Rome and the mosque of Little Hagia Sophia in Constantinople, formerly the Church of the Saints Sergius and Bacchus.

The vault of the octagonal hall can be defined as a sort of mixtilinear octagonal cloister vault.

The ancient architect decided to customize previous achievements by Severus and Celer in designing and constructive matters: the dome is the product of a shape blending as well as Nero's Domus Aurea where a regular octagon turns into a regular polygon of sixteen sides.

At the Small Baths a first octagonal mixtilinear silhouette (the drum) gradually fades into an intermediate square, and then it finally becomes a regular dodecagon on the top of the roofing approximating a spherical cap through a set of twelve ruled surfaces.

A complex problem is how they achieved the complex intersection in constructive terms that appears near to the spring-point of the cupola generating a double curvature curve, very unusual in Roman building design as it is the outcome of the intersection of two cylinders. However this fact is strictly related to the theories included in geometrical treatise as the one "*On vaulting*" by Heron. The thick mortar laid upon the intrados did not allow the understanding of how the ancient builders could have achieved such a peculiar shape without apparent discontinuities.

The next phase of this study will deal with the geometrical analysis of the Piazza d'Oro Vestibule, besides the reconstruction hypothesis of other two buildings belonging to the Hadrian's Villa, the Reverse-Curve pavilion of the Accademia and the southern triclinium/nymphaeum of the Piazza d'Oro.

The outcome from Small Baths, both in terms of methodology and knowledge on ancient designing techniques for vaulted spaces, should be applied to these last two halls with the aim of having a complete picture of tent-like cupolas at disposal.

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