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GEOMETRY OF WALL DEGRADATION: MEASURING AND VISUALISING IMPACT CRATERS IN THE NORTHERN WALLS OF POMPEII

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Abstract

The paper presents the methodology used to digitally document the cavities preserved in the northern city wall of ancient Pompeii, to obtain a detailed documentation of the visible ballistic traces attributed to the impact of Roman artillery during the siege of Sulla in 89 BC, in the area from the Vesuvio to the Ercolano Gate. Building on previous studies by the team, authors intend to illustrate the workflow carried out to analyse the morphological characteristics of the existing traces by means of reality-based 3D digitsl models. These virtual replicas are intended to improve the knowledge of selected case studies due to their high relevance. In addition, some stone projectiles still preserved in the *Antiquarium* will be analysed. The final objective is to define a suitable approach for quantifying the terminal ballistics of Roman artillery based on conclusive evidence, thus allowing not only the virtual, but also the physical reconstruction of the launching weapons used in the war event.

Keywords

City walls of Ancient Pompeii; Documentation of Cultural Heritage; Reality-based 3D Digital Models; Negative/Positive Virtual Moulds; Reverse Modelling procedures

1. *Introduction*

The peculiar circumstances that still allow the documentation of anthropic origin of the cavities that are present in large numbers on the northern fortifications of the archaeological site of Pompeii (Fig. 1), lead to the SCORPiò-NIDI project.[1] The marks generated by the collision of projectiles that allegedly pulverised a certain area of the surface of the city wall between Vesuvio Gate and Ercolano Gate (Van Buren, 1925, 1932), incontrovertible evidence of the deadliness of the Roman artillery used in 89 BC to besiege the city of Pompeii, were also confirmed by the archaeologist Amedeo Maiuri (1886-1963), who completed the excavations in the late 20s (Maiuri, 1930, 1944). The defensive wall, enclosing with its towers the *Cornelia Veneria Pompeianorum* colony (Pesando, 2015; Cicerone, *Pro Sulla*, 61-62), which became a colony of the Empire with the victory of Sulla, was then buried in 79 AD by the volcanic eruption.

Despite the degradation occurred due to environmental and meteorological factors (temperature, humidity, etc.) and phenological events (development of moss and lichens) in the aftermath of the excavations started in the 18th century, the current state of preservation of the city wall still allows the acquisition of accurate data of the original damage, suitable for describing (i.e. calculating and demostrating) the impact force of the projectiles hurled.

Among those who have drawn attention to the exceptional nature of the ballistic evidence represented by the cavities (Fig. 2), which can be detected in their material aspects and therefore scientifically examined using up-to-date methods

[1] *SCORPiò-NIDI. Comparative Analysis and Certified Reconstructions for a correct experimental archaeology: Roman Scorpions and Ballistae for the Imperial mechanical culture, origin of European identity. Governance policy for the development and sustainable fruition of Cultural Heritage.* Bando Prin 2022 – Decreto Direttoriale n. 104 (02-02-2022), Prot. 20222RJE32, Settore ERC SH5 "Cultures and Cultural Production", admission to MIUR funding DR 10790/2023. UNICAMPANIA Team: Principal investigator Full Professor Adriana Rossi; Associate Professor Sara Gonizzi Barsanti; Research Fellow Silvia Bertacchi; Ph.D. Student Claudio Formicola. The 24-month project, started in November 2023 and still ongoing, is aimed at reconstructing virtual and physical working replicas of Roman war machines of the Republican and Imperial periods, based on an in-depth multidisciplinary study on the impact craters of the northern city walls of the unearthed city.

Fig. 1: The northern city walls of Pompeii, seen from the Vesuvio Gate towards Tower X. Credits: A. Rossi

and devices, mention should be made of the experimental study by the engineer Flavio Russo, collaborator to the Historical Office of the Army General Staff, author of series of volumes dedicated to technology transfer encouraged by military research over time (F. Russo, 2009).

It is worth mentioning that upon his request, in the early 2000s officials of the Pompeii Superintendency asked the offices in charge and obtained permission to take physical silicone moulds of the major craters (F. Russo & Russo, 2005, p. 62; Rossi, 2024, fig. 5), some of which casts are currently on display in the Archaeological Area of Saepinum, Altilia (CB).

Fig. 2: Picture of one of the marks still visible on the stone ashlars of the city walls caused by a stone ball hurled by the besieging army. Credits: S. Gonizzi Barsanti

The results of this experimental research, only partially disclosed, were collected and organically developed in a recent publication (Rossi, 2024), that argued for the need for a digital reality-based survey, optimal for future reference, with the aim of studying, preserving and disseminating, in step with the times, this important evidence with the ultimate goal of a renewed image of the famous unearthed city.

In this paper we will therefore present the documentation operations carried out by the team, with the aim of:

i) collecting archaeological, historical, architectural and military information on siege defence techniques;

ii) tracing highly relevant examples of stone ball impacts, and possibly discern the damage supposedly caused by other types of coeval weapons on the city wall between the Vesuvio and Ercolano Gate;

iii) identifying a criterion for classifying impact cavities of different diameters and shapes;

iv) documenting and analysing the morphometric and radiometric characteristics of the existing impact cavities through accurate high-resolution reality-based 3D digital replicas, collected by means of non-contact image-based techniques, and derive critical information;

v) exchanging interoperable copies, simplified but consistent with the reality-based model, converted into numerical models to simulate the dynamic behaviour and thus obtain the parameters required for the subsequent calculation of the shooting power;

vi) finally, approaching a communicative (informative and constructive) system to generate Digital Twins (DT) functional for the purposes of the SCORPiò-NIDI project and, more generally, to populate a dedicated functional platform to collate models and knowledge of the subject, in progress and for future reference.

The paper is structured as follows: a concise presentation to contextualise the subject of the study and the research up to date (paragraph 2); then the methods and devices used to document the impact traces are analysed (paragraph 3), starting with a general survey of the area of interest of the city wall and then moving on to a detailed study of the selected case studies; the analysis of the geometric characteristics of the different types of holes documented and of the surviving projectiles still preserved in the *Antiquarium* of the Pompeii Archaeological Park will complete the analytical phase and the possible outputs (paragraph 4). The conclusions (paragraph 5) will summarise the main results of the study and the future steps to be taken to achieve further objectives of the general research project and, possibly, to increase the impact of the study on the knowledge of siege warfare.

2. The ballistic craters and research done to date

The walls of Pompeii represent a case study of great interest to archaeologists and military historians, who have studied theoretical and operational aspects, such as the analysis of the phases of transformation of the city walls, and the construction techniques and properties of the materials used, or the physical survey of the defensive wall complex (Sogliano, 1917; Maiuri, 1930, 1943; V. Russo, 2013; Anniboletti, 2016; Fabbri, 2016, 2021).[2]

The particular conditions of preservation of the walls, hidden under the layer of lapilli due to the eruption of the Vesuvius in 79 AD, until the rediscovery of the ancient city, can allow researchers to obtain an important document on the effects and, consequently, the power of the Roman elastic launching weapons of the time, i.e. the Republican-Imperial period.

Excavations began in the mid-18th century and the city was uncovered in several stages, but it was the archaeological research by Amedeo Maiuri in the twentieth century that brought to light stretches of the city walls, increasing knowledge of the history of the fortifications and their evolution (Maiuri, 1930).

The interesting traces in the northern walls of the city had already been recognised as possible ballistic impacts from Sulla's siege, which historians date back to the 1st century BC (Maiuri, 1943, p. 148). During the Social War, in fact, several sections of the city walls were affected by the damage caused by the siege artillery.

In the section between Porta Vesuvio and Porta Ercolano, many circular holes caused by the projectiles can still be seen, especially near the latter and in greater numbers on the E and W sides of Tower X. It is no coincidence that most of the traces can be found in the area built with ashlars, because it was considered to be the weakest area of the defensive perimeter due to the slope where the assault concentrated.

[2] Scholars recognise a succession of different architectural phases in the fortifications of Pompeii, which can be roughly summarised as follows: i) Archaic phase, with a defensive system made of blocks of alluvial sandstone from the nearby Sarno river and effusive lava rock from the Pompeii hinterland (second quarter of the 6th century BC to the beginning of the 5th century BC); ii) Pre-Samnitic phase, with a double wall of quadrangular slabs of Sarno limestone (5th century BC); iii) first Samnitic phase with replacement by a masonry wall of regular limestone blocks crowned with battlements, extending over the current perimeter; there was probably an external moat parallel to the walls to defend the lower and less protected areas of the city, i.e. the northern one (late 4th century BC); iv) second Samnitic phase, with the raising of the wall in tufa blocks and a parallel second inner wall with agger embankment (late 3rd century BC); v) late Samnitic phase, with the construction of 12 towers placed above the wall with lava rocks inserts, as well as the reconstruction of large parts of the walls in *opus incertum* (last quarter of the 2nd century BC); vi) phase following the siege, with the restoration of the damage to the fortification using various materials and construction methods, and also the remodelling of the level of the outer pomerium through the gradual softening of the slope and the dumping of building materials following the earthquake of 62 AD. For further details and for the classification of the construction phases on the preserved sections and for the excavation phases, see in particular the report by Sogliano and Maiuri, and for the more recent opinion on the evolution of the fortified walls and their conservation, see the research by V. Russo, Anniboletti and Fabbri.

Another reason for this could be due to the fact that other parts of the walls were subject to restoration and altered in uncertain work as a result of later repairs or made of volcanic ejecta with a less regular wall texture, on which it was impossible even at that time to trace such impact marks.

Yet in 1925 the archaeologist Van Buren (1878-1968) documents in the period photo (Fig. 3) these marks of ballistic impact, found in large numbers along a portion of the walls extending some 30 metres to the west of the Vesuvio Gate and for some 8 metres to the east of Ercolano Gate (Van Buren, 1932, p. 14).[3]

According to the scholar, the curving holes were probably due to the impact of balls from ballistae and shot from slings, which collision would have pulverised a certain area of the surface of the stones of the wall based on the force of impact (Van Buren, 1925, pp. 110–111). Some additional observation to demonstrate such theory take into consideration the absence of chiselling or tool work, as well as their location on different parts of the face of the stones so that they are not to be interpreted as cramps or dowel holes (Della Corte, 1913; Van Buren, 1925, p. 110).[4] Other hypotheses as cause for the cavities have also been put forward, which have been rejected in favour of documented siege techniques that explain well how the craters in the stone can be considered as missed blows due to incorrect trajectory (too low elevation) or to reduced charging (too weak shot) of the lauching weapons. The actual target was probably the temporary wooden shielding devices erected on the battlements to hide the defenders (Iosephus, 1989; F. Russo & Russo, 2005).

Fig. 3: Picture of the bombardment marks caused by the Sullan troops during the assault to the city in the vicinity of Vesuvio Gate. Year 1925 ca. Credits: Van Buren, 1925, Plate 60, I.

Returning to the ballistic traces found at the site, Van Buren divides the marks of the Sullan bombardment into three classes, due to their diameter (large, intermediate, and small, respectively for 14 cm, 4 to 8 cm, and 1 to 2 cm).

Still, in the historical photo included in Van Buren's text, we can easily recognise them in comparison with the actual photographic campaign (November 2023-February 2024) (Fig. 4).

Fig. 4: Comparison between the state of preservation in the 20s (photo on the top, year 1925) and the actual 3D surface (3D model on the bottom, February 2024) of the city wall; detail detacting the differences on the left tufa ashlar, nowadays partially missing probably due to the impacts visible in the photo that contributed to deteriorate the stone. Credits: Top, as Fig. 3; Bottom, model by S. Bertacchi.

[3] The scholar Van Buren, some years later, also counts the marks of some large missiles still visible in the stretch that is faced with Sarno masonry near the Torre di Mercurio (Tower XI), while a number due to small projectiles are visible in the portion of wall that is faced with tufa blocks to the west of that tower.

[4] Van Buren mentions that according to Dr. M. Della Corte of the Administration of the Excavations at Pompeii, the observed marks were caused by some instrument of percussion, probably an aries with a metallic head impact, although it seems unlikely that attempts would be made to break down the impenetrable walls with such systems.

The degradation that has taken place over the last two centuries once again points out the need for complete digital documentation of the state of preservation, threatened by further devastating natural disasters and not least human negligence.

3. Methods and technologies for advanced documentation

In recent decades, the diffusion of digital acquisition techniques, even in the cultural heritage sector, offers many advantages, demonstrating the potential of novel tools to share scientific results among scholars in a fast and efficient way, also through online virtual platforms implemented for the intelligent management of all the data collected, based on 3D models that allow for a better visualisation and dissemination of knowledge (Bertacchi et al., 2018).

In particular, the existence of accurate and rapidly updatable 3D digital models to reflect changes in their physical replicas is one of the most fruitful areas to date. Therefore, it is necessary to create a reliable digital ecosystem for the critical use of the various data acquired, which does not remain a mere heavy and unstructured documentation – difficult to access and hardly usable – but acquires the value of a virtual simulacrum. The latter, containing not only information describing the physical shape and appearance, but also linking a series of information of different nature for the promotion and use of the specific topic, i.e. the Roman mechanical culture of the Imperial Age. For this reason, we need reliable and certified three-dimensional digital replicas, which are now considered indispensable for a complete and effective description of ancient objects, making it possible to store morphometric and surface information that is valuable for monitoring over time. Moreover, to be used as an access interface to a virtual information system and for advanced representation, ranging from the 3D model itself, which replicates the object of study at a scale of 1:1, to the associated two-dimensional graphical elaborations extracted as raster or vector files, to provide scholars and experts with a set of information useful in various disciplines.

In general, Cultural Heritage is subject to regulations and protection measures, so for conservation reasons the use of non-contact devices is currently preferred in case of heritage surveying. Additionally, digital techniques of acquisition and advanced representation are favoured over traditional drawing techniques due to their potential to reproduce in greater detail and with multiple outputs complex objects. This is valid especially when dealing with irregular surfaces such as deteriorated stonework (Fig. 5), where articulated shapes or organic morphologies would be difficult to represent properly with only traditional two-dimensional representation or be oversimplified by the operator as in the past.

In fact, the first objective is to obtain a complete and accurate description of the surface to document and, above all, to recognise the nature of the traces found on the stone, in other words, to avoid the risk of altering, through incorrect intervention, important evidence of the past that every ancient structure carries. Hence, it becomes essential to eliminate "false positives," which may be due to the very nature of the construction material used (holes and cavities due to porosity and to the constitution of the material) and degradation of a natural type intervened with, for example, exfoliating of the material for atmospheric agents or breakage/fall of parts. These should be discerned from signs of workmanship (depending on the tool used to cut the stone, etc.) and above all from possible intentional damage inflicted on ancient masonry during wars and sieges.

Prodromic is therefore the survey project that aims to identify the most appropriate technological solution for recording the morphometric characteristics of the artefact, thus facilitating its reliable description with the precision and resolution established a priori.

Regarding the research object, the team carried out the following documentation: a) a terrestrial laser scanner survey of the city walls

Fig. 5: One of the marks still visible on the tufa ashlars of the city walls caused by a stone ball hurled by the besieging army. Credits: S. Gonizzi Barsanti

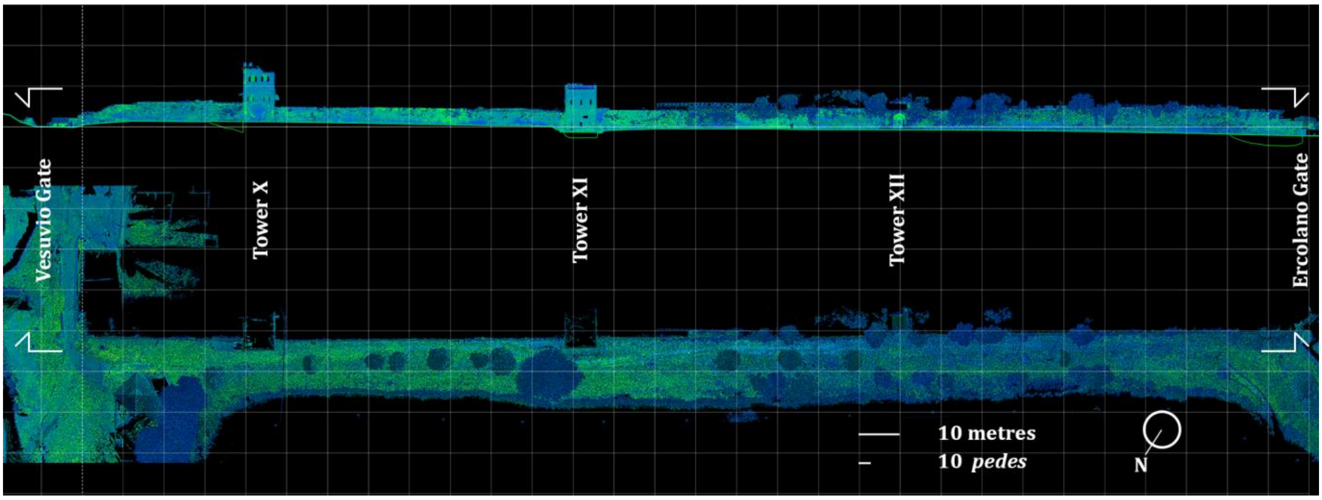

Fig. 6: Front section and plan view of the point cloud from TLS survey of the northern fortification in Pompeii. Total 374 million points. Credits: S. Bertacchi

between the northern gates for general reference, identification and contextualisation of the case studies, b) close-range photogrammetric survey of some ballistic marks, considering different sizes of impacts, to study its dimensional and surface characteristics.

3.1 General reference activities

After a preliminary photographic campaign for general reference, an overall documentation specifically planned to describe properly the present-day orography of the site (size and length of the complex and construction elements, inclination of the city walls, height of the surviving parts and accesses with respect to the current ground level, etc.) has been carried out before any detailed acquisition, in order to support with reliable metric information the correct spatial localisation of the specific case studies and the recording and interpretation of ancient architectural elements (Gonizzi Barsanti, Remondino, & Visintini, 2013; Benedetta Adembri, Cipriani, Fantini, & Bertacchi, 2015; B. Adembri et al., 2016; Ottati, Bertacchi, & Adembri, 2019).

The general documentation was therefore carried out by a Terrestrial Laser Scanner (TLS) survey,[5] whose reliability is guaranteed by a topographical reference network. The nature of the object, a ca. 300-metre extended defensive wall, interrupted by the remains of three towers (X-XI-XII), and with only these few elements taller than the walls, allowed an initial series of scan stations at the ground level of the walkway at the base of the city walls (Fig. 6). Far from returning a complete image of its overall thickness and structure, which has already been the subject of extensive surveys for other documentary purposes, this documentation focuses at this initial stage of the project only on the exterior façade of the fortification, considered sufficient for our purpose and possibly to be integrated with different devices in the future, thus providing the general reference 3D digital model for the external wall surface starting from *Castellum Aquae* and between the Vesuvio and Ercolano Gates, this latter excluded (Fig. 7).

After the in-situ acquisition, single scans underwent a standard processing workflow, including noise reduction, erasing of unnecessary areas, and registration in a single reference system by means of Leica Geosystems Cyclone software. For practical reasons that are functional to our purpose, the general model has been oriented in a local reference system for technical elaborations, aligning the external façade to the x axis and locating the origin (point 0,0,0,) at the visible earth connection of the northern wall from the corner with Vesuvio Gate, considered as the best orientation to obtain general fully descriptive orthoimages. This user coordinate system will be

[5] TLS survey activities were carried out in January-February 2024, consisting of 45 scan stations with Leica ScanStation P30 (time-of-flight technology, laser class 1) with these main specifications: 3D position accuracy of single measurement 3 mm at 50 m; horizontal 360° and vertical 290° field-of-view; scan rate up to 1 million points per second; resolution value to medium; average range 120 m, accepted range 50 m.

shared by all the digital models created through applied surveying methodologies.

When this step has been completed, the raw data from TLS has been non-automatically subdivided according to a semantic partition, dividing the fortification complex (city walls, towers and gates), from natural elements (ground-level walkway and slope, street, trees and vegetation) or other elements. Since the wall surface is the area of greatest interest for the project and for identifying case studies for design analysis, a further subdivision into levels of detail is expected, which can include sectors between the towers, distinguished by materials (limestone, tuff, lava rocks…) and construction systems (regular construction with squared stone blocks of equal height or using irregularly shaped and randomly placed lava rocks walls), up to single ashlars to locate the ballistics marks. With respect to individual highly detailed models, for user-friendly visualisation reasons, when on display mode pivot point will be automatically set in the baricentre.

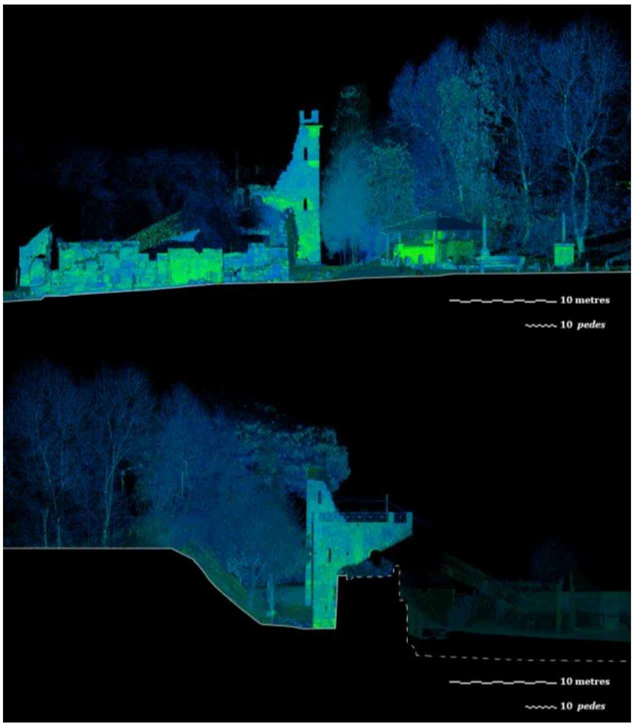

Fig. 7: Two views of the TLS point cloud: on the top section along the street of Vesuvio Gate; on the bottom section on the western side of tower XI. Credits: S. Bertacchi

After editing and meshing operations (close holes, optimisation...), separated into several parts for processing due to the detail of the models, the polygonal mesh of the surveyed surface is complete and integrated with close-range photogrammetric survey campaigns for enhancing the radiometric aspect,[6] i.e. the texture of apparent colour (Fig. 8). The combined use of laser scanning and photogrammetric surveying made it possible to exploit the advantages of each technique while obtaining the best result in terms of completeness of information (actual state of preservation and radiometric aspect) besides metric reliability and geometric data.

The final model will have then two versions with scalable resolution depending on the level of detail based on project purposes, a master model (high-quality for research purposes) and a lighter version, optimised by means of a manual retopology process, which can be efficiently used for demonstration and visualisation-only purposes or for linking various sources of information.

3.2 Photogrammetric focus on impact craters

Precision documentation activities deal with with the surface survey of the impact craters, recording their geometric characteristics and the depth of penetration of these ballistic impacts to identify, classify and describe the existing traces that have survived to the present day.

In fact, it is necessary to determine the size of the holes and their shape, which in the case of the spherical ones is quite obvious, given their size (radius range from around 4 to 8 cm), the depth of the depressions (up to even more than 10 cm), as well as the angle of incidence of the shots and the height of the traces, bearing in mind, however, that the current ground level is the result of the archaeological excavation and in all probability not at the same level as at the time of the siege, essential data when considering the parameters of the launching system and its possible reconstruction (Vitruvius, 1997, bk. X, 11).[7]

[6] For the general photogrammetric campaign, a Nikon 5200 (24Mpix APS-C size CMOS sensor and focal length lens) was used. For the detailed acquisition APS-C Canon EOS 60D, EF-S60mm f/2.8 Macro USM (60mm), and a full frame Canon EOS 5D Mark III, coupled with 20 mm and 16-35 mm lens.

[7] From these data, experts can calculate the structure of the ballistae, considering that the Roman architect Vitruvius in his tenth book gives important instructions on geometrical proportions based on the *modiolo* and the correspondences between the weights of the projectiles, expressed in *libra*, and the diameters of the elastic strands, expressed in *digitus*. This relationship makes it possible to reconstruct the material characteristics of the weapons, given the weight of the projectile, obtained by the reverse process from the diameter

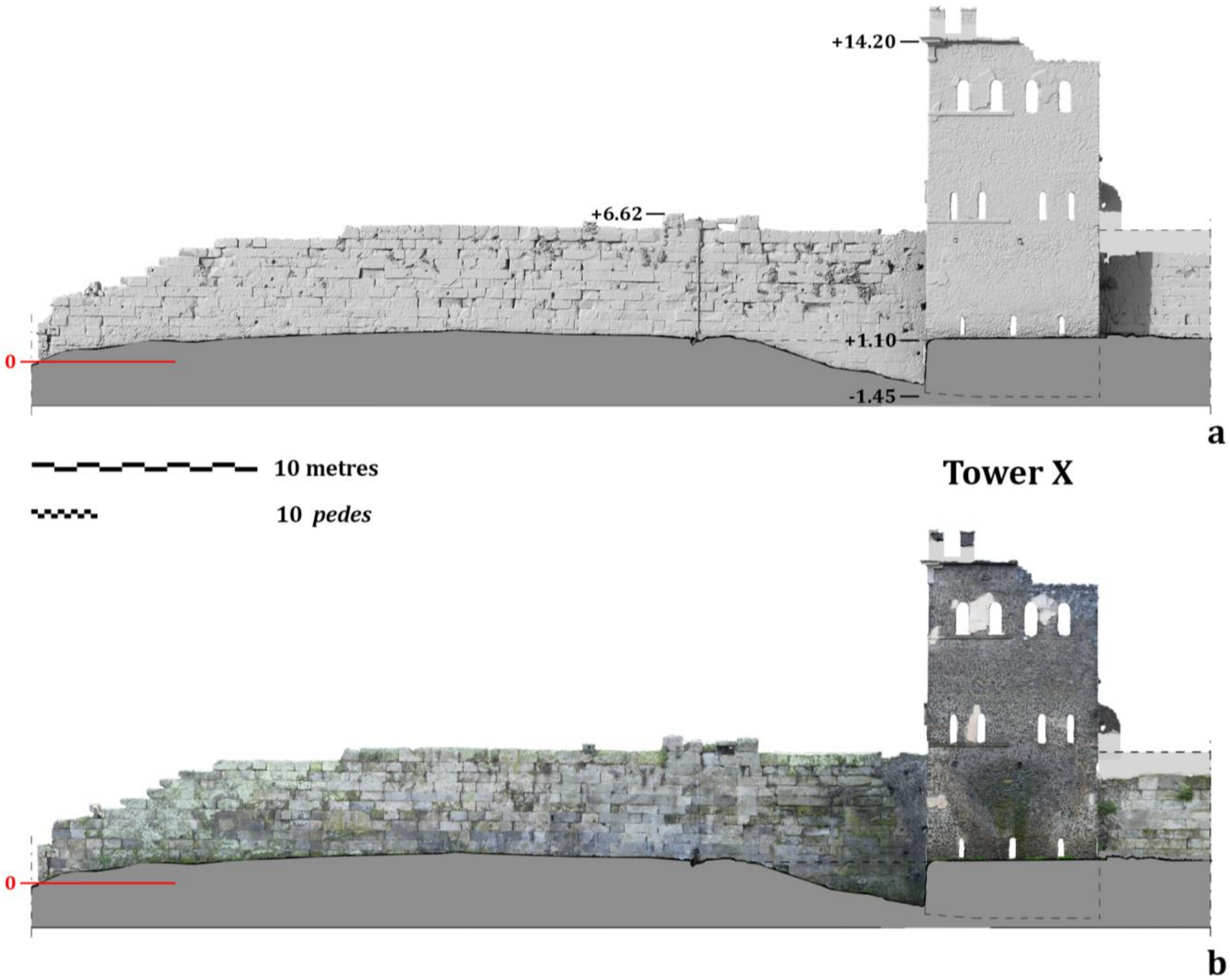

Fig. 8: Front view of a portion of the polygonal mesh of the fortification, east of Tower X. a) polygonal mesh from TLS data; b) textured photogrammetric model. Credits: S. Bertacchi

The present level of the ground actually covers a part of the ancient wall: this can be seen by the partial covering of a part of the rows of the stone elements that were probably at the level of the side sortie of the tower in ancient times, then covered by the eruption and only partially excavated or covered/filled over time, an occurence also mentioned by Maiuri (1960). This fact is also confirmed by the ground level of the towers, today about 2.5 metres below the level of the traces, whereas at that time there must have been a moat and a lower level, the height of which probably changed first as a result of the defensive walls falling into disuse after the siege, then as a result of the destruction caused by the seismic event in the middle of the 1st century AD, and finally as a result of the eruption in 79 AD.

This procedure of morphological capture is performed via non-contact technologies recommended for preservation reasons, such as close-range photogrammetry. Current photogrammetric surveying technologies, in fact, due to their speed, cost-effectiveness and achievable accuracy on optically cooperating surfaces, are among the most competitive technique for the analysis of such artefacts.

of the hole caused by the impact or directly from the projectiles (stone balls) still preserved in the storage of the archaeological site. Given the size and density of the projectile, the dynamics of the impact and the damage (ball marks) will be reconstructed, also taking into account that the material of the walls, characterised for this purpose by mechanical tests. This result will be used to reconstruct the trajectories of the projectiles (numerical finite element analysis) and thus to analyse the launching machines used and to reconstruct them in digital form, functionally and dimensionally, in every component and detail.

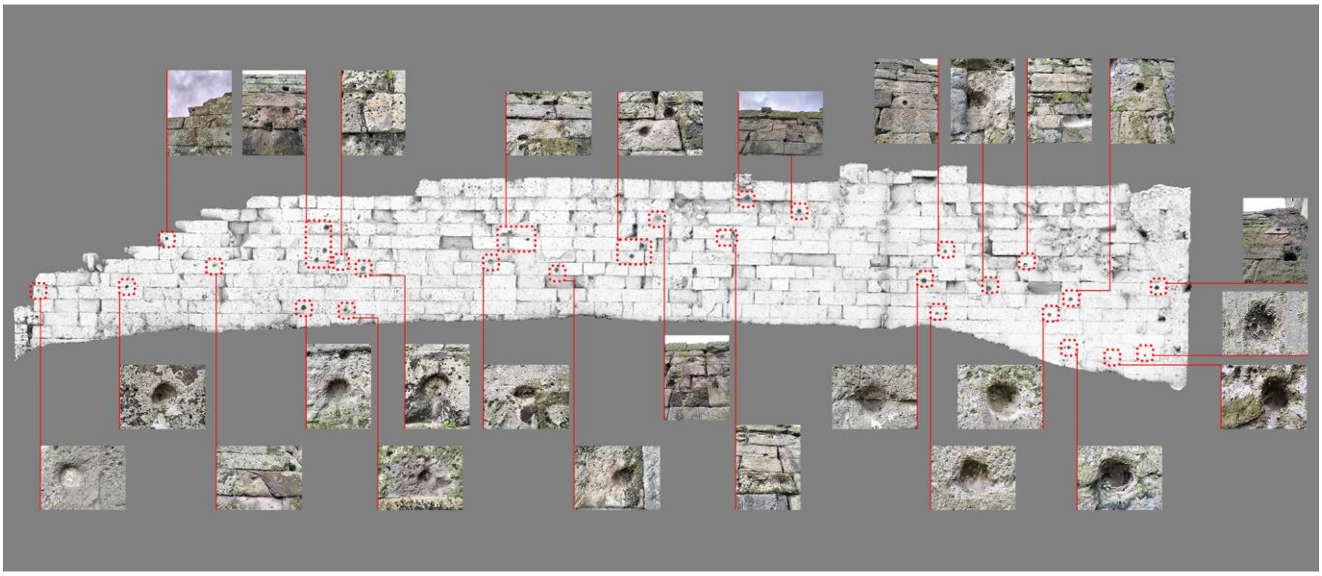

Fig. 9: Location and pictures of some of the ballistic marks (stone balls) in the wall adjoining the Vesuvio Gate. Photo credits: S. Gonizzi Barsanti

With current surveying technologies, photogrammetric systems, thanks to image matching algorithms such as Structure from Motion (SfM), allow the surface to be captured with accuracy and flexibility, due to the presence of homogeneously distributed textures that facilitate the internal orientation of the camera.

A partially automated procedure, customised for the case study, involves the image processing according to a workflow that identifies features on individual images, then correlates them with respect to the image sequence, and finally performs the resection of the cameras in the scene through bundle adjustment operations. Given a suitably structured a priori positioning network of the individual shots, the density and quality of the point clouds are comparable to those obtained by active sensors. By transforming these data into polygonal meshes with applied chromatic information, conveyed by mapping in the parameter space of the mosaic of photographic shots, an advanced 3D object is obtained from which a series of elaborations can be derived.

However, the image-based approach to documentation requires some important precautions to ensure that the final digital model is metrically and morphologically complete and reliable. In particular, the following parameters related to the specific camera must be determined a priori before starting the detailed survey:

1. Ground Sample Distance (GSD): in order to obtain the appropriate resolution of the digital models to be determined a priori, depending on the size of the sensor used, the logistics of the site and, therefore, the possibility of close range for acquisition;

2. Depth of field (DoF): this parameter, closely linked to the previous one, must be set so that the sharpness of the image (a necessary condition for obtaining a correct reconstruction of the scene by SfM) is equal to or greater than the depth of the object to be documented. In the case of hollow areas (e.g. the hole) with respect to the mid-plane of the surface, this parameter is relevant to obtain images with acceptable sharpness, avoiding blurred parts that entail noise and geometric problems during processing;

3. Sufficient horizontal/vertical overlap in the photographic frames to properly document the surface without missing data in any area, also taking into account the case of protrusions or cavities, for which it is necessary to obtain inclined and not only orthogonal images;

4. Acceptable illumination of the deep recesses in the wall. As a matter of fact, passive sensor techniques suffer from lack of lightning, making it impossible to detect the deepest concavities without an illuminator aimed at significantly reducing the occlusion effect and thus preventing the presence of shadows that determines a low-quality 3D reconstruction;

5. Inclusion in the scene of a colour balance reference and a metric reference, for correct scaling of the digital model.

The photogrammetric campaigns therefore included a series of case studies (Fig. 9), initially selected according to some main characteristics: 1) the type and dimension/depth of the projectiles that caused the damage, 2) the accessibility of the case study to the operator with the device (easily accessible for documentation).

3.3 3D tools for visualisation enhancement

Once the 3D digital copy of the stone surface and its traces has been obtained with the required resolution, and following intelligent optimisation to correct its geometric errors and topological anomalies, the polygonal model is congruous to be used for multiple survey operations (controlling its model confidence), either within the software for virtual 3D model management and visualisation, or through two-dimensional elaborations useful for the purpose and available for restoration or preservation operations by the operators.

This model, as already mentioned, has a twofold purpose: on the one hand to provide the possibility of a preliminary screening to identify, classify and locate recognisable impact craters, and on the other hand to correctly locate the elected case studies within the city walls system for subsequent analyses on height position, shot incidence, and possible location statistics.

Firstly, it is considered to be a valuable support for observing impacts: on direct observation those too high would remain difficult to see and are in any case subject to recognition by an experienced external operator. Thanks to the digital model, it is possible to exploit the potential of software for management and visualisation in a virtual environment with different options for analysis, not least of which is the improvement of the perception of the shape with the visualisation tools that characterise the world of digital representation thanks to different illumination systems or the application of specific maps for restoring detail, which emphasise the geometry and increase the readability for the user using the model for study purposes (Pintus et al., 2016).

Concerning the possibilities of graphic visualization and representation, one can enumerate the most frequently used ones for this phase of the project (Fig. 10):

1. *deviation* of the mesh in false colours with respect to virtual planes chosen by the operator to emphasise cavities and protuberances (Fig. 10-c). The mesh deviation shows distances from a

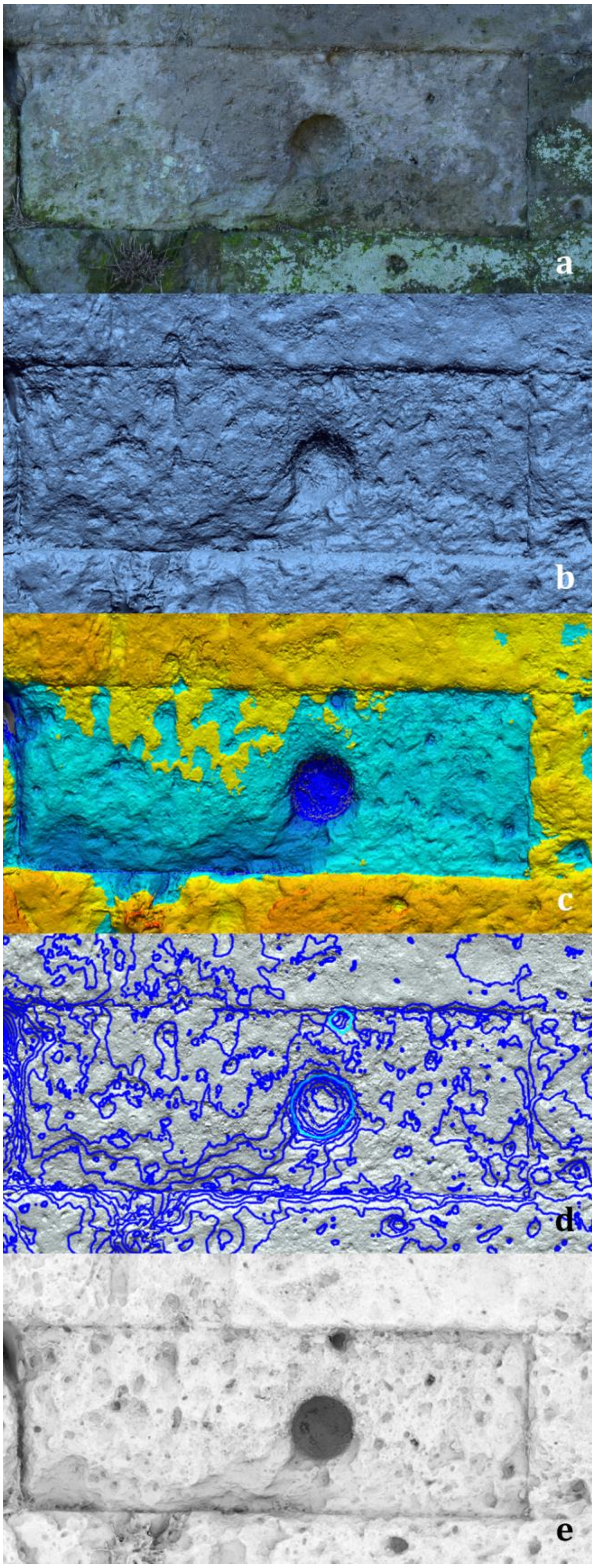

Fig. 10: Tools for analysing the 3D model of a damaged stone ashlar: a) textured 3D digital model; b) high-resolution polygonal mesh; c) false colour representation of the mesh deviation from a reference plane; d) contour lines; e) occlusion map. Credits: S. Bertacchi

vertical reference plane with a customisable shade of colours, that can change shade towards blue for increasing depth, and towards red for increasing protrusion in a selected range, graphically emphasising the non-flat aspect of the wall. This tool works better locally, for example by considering a single ashlar and related average plane on the surface, since the masonry is irregular and cannot be referred to with a single plane; moreover, because of the wall inclination it is difficult and very approximate to consider a single plane for the entire section of the city wall. On the contrary, it gives better control of the surface course, proving very useful;

2. *contour lines*. Sectioning the model with various planes set by the operator at predefined intervals lines (e.g., every cm or even mm), is an easy way to create polylines enhancing the graphical representation of the geometric shape and helping the dimensional interpretation of the irregular geometry of marks (Fig. 10-d);

3. *special two-dimensional maps*. Visualisation in false colours or with specific maps is paramount for understanding the complexity of the object through graphical representation. As an instance, occlusion maps (Fig. 10-e), i.e. textures associated with the model that contain precomputed shading information, critically help to distinguish hollow areas, darker than others. These latter can be used not only for visual verification, but also processed by means of feature recognition software on high-resolution images (employed in other field of study, e.g. the open source software Fiji) (Fig. 11), for example by automating the process of identifying features of interest, highlighting holes of greater depth, similar diameter, or traces tending towards regularity or even circularity, to which further attention should be considered.

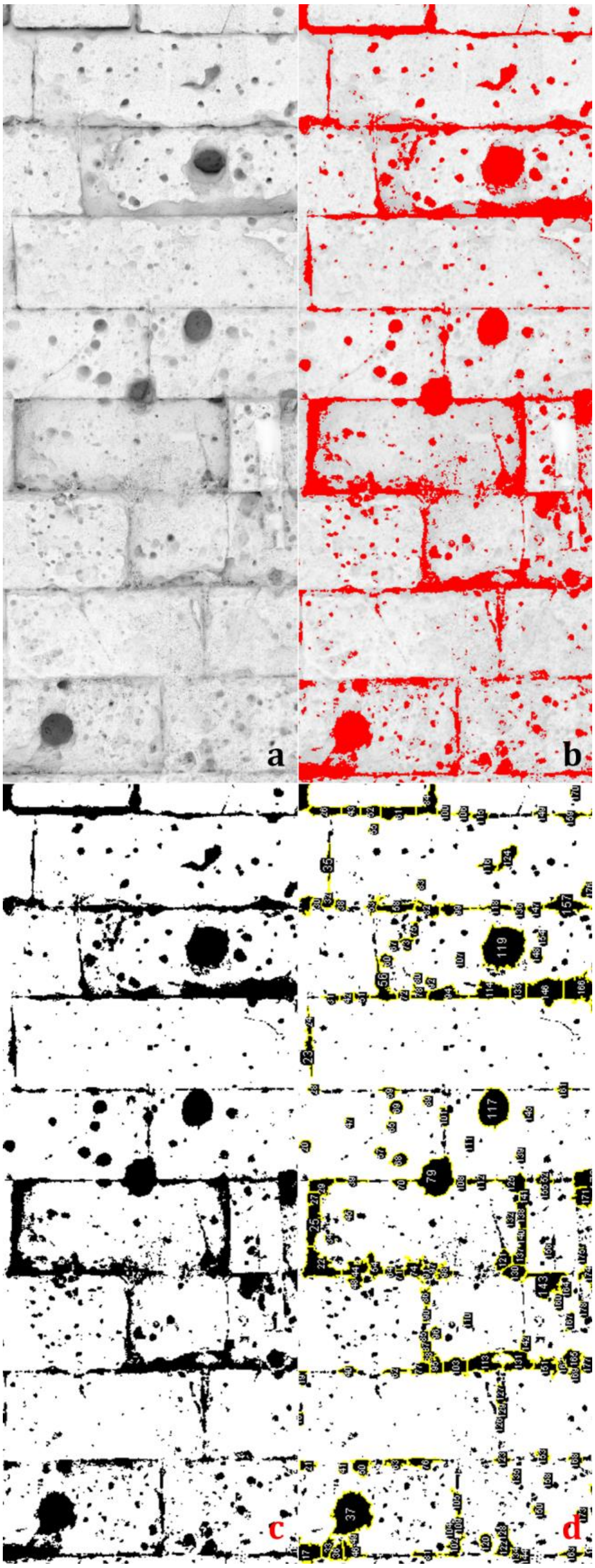

Fig. 11: High-resolution rendering of a section of the city wall presenting ballistic marks: a) occlusion map to be elaborated with Fiji feature recognition software; b) manually adjusting threshold (highlight of deeper holes in red); c) search for circular shapes; d) analysis of particles for evaluating circularity. Credits: S. Bertacchi

4. Outputs

4.1 Creation of virtual moulds

Still, it should be noted that as long as the model is obtained consistently with the described workflow, the related digital copy will correspond to the physical object, moreover, without any need of physical contact with the surface. Precisely for this reason digital ones are preferred to traditional moulding techniques, employed in the past (i.e. by applying silicone resins in several layers on successive days, filled with scagliola and then removed from the support after drying), because of the possibility of damaging the surfaces by tearing ancient deposits from the surfaces (i.e. plaster) when removing the dried cast that cannot be recovered or restored.

It is also evident that physical casts are subject to important issues, primarily cracks, deformation, and shrinkage over time, as well as the deterioration of the constitutive material, not to mention the fact that they provide limited documentation, as they lack any general reference of the real location, unlike digital ones, that can be referred to a coordinate system in space.

In terms of surface characteristics, thanks to reverse modelling process on the model, one can obtain virtual casts to extrapolate dimensional and geometric information, such as shape and depth, and to efficiently represent the cavities (Fig. 12). From the actual model of the craters in the city wall (the negative mould), a positive virtual mould can be extrapolated by simply flipping the normals of the measured digital surface.

The available polygonal mesh, when subject to automatic detection for regions, allows automatic identification of simplified geometric primitives (cylindrical, spherical, etc.), that best fit the selected parts, helping scholars to hypothesise surface or solid shapes that could coincide with the object that caused the impact (tools, projectiles, etc.).

At this point, the automatic extrapolation of the theoretical shapes, up to the metric study of the positive cast, can follow customised procedures, depending on the subject, to improve the acquired shape by eliminating irrelevant geometric peculiarities and proceeding with controlled smoothing of the polygonal mesh to regionalise the cavity and obtain solid shapes by manual selection of the most significant regions detected.

4.2 Analysis of projectiles (stone balls)

It is well known that the elastic-torsional artillery used by the Romans was derived from Hellenistic prototypes, and there have been many studies on the ancient launchers, most of them based on theoretical considerations rather than on physical evidence of the effects of the shots (Vitruvius, 1997; F. Russo & Russo, 2005, p. 78).[8] In a nutshell, it could be said that these projectile

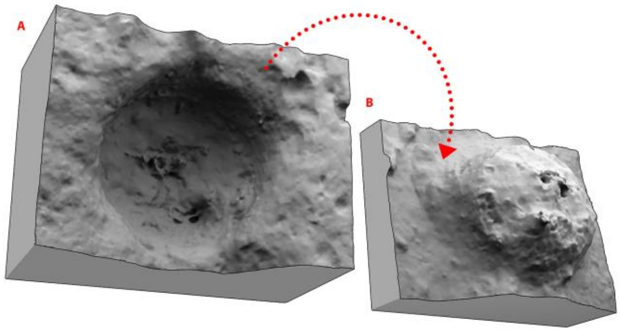

Fig. 12: Virtual cast of one of the cavities: a) wall stone surface (negative mould) and b) reversed mesh (positive mould). Credits: S. Bertacchi

weapons, which were available to the Romans at least until the second half of the 3rd century BC (F. Russo, 2009), were characterised by the type of projectile, which could be either stone balls thrown by ballistae, or metal-tipped bolts, the latter launched by catapults (also called scorpions) to throw bolts at a distant target (F. Russo & Russo, 2005, pp. 78–96).

Obviously, the throwing of penetrating bodies, in the case of blunt objects such as stones or metal-tipped arrows, was intended to respectively damage defensive structures, such as the wooden palisade protecting the soldiers deployed at the top of the city wall, or to injure besieged defenders with shots from a distance, the range of which was directly proportional to the power of the weapon. This artillery did not, in fact, have the destructive power to damage stone walls with thick reinforcements, so the impressions made in the city walls most likely represent unsuccessful shots rather than deliberate attacks to affect the structural integrity of the thick wall (F. Russo & Russo, 2005, pp. 90–92).[9]

The ballistae (F. Russo, 2004; for the range of 180 m ca. see Molari, Maraldi, Angelini, Bignami, & Lionello, 2012; Rossi & Palmieri, 2020; Rossi, Palmieri, & Barsanti, 2022), which differed for the arm movements, were divided into *eutitone* (straight tension, rotating outwards from their fulcrum) and *palintone* (reverse tension, rotating inwards from their fulcrum) (Heron, 1572). According to some scholars, *palintone* weapons could hurl stone balls at a speed of less than 100 m/s (F. Russo & Russo, 2005, p. 90), more

[8] For torsional artillery and virtual reconstruction of models, see the research by Flavio and Ferruccio Russo, especially for the reconstruction of catapults, ballistae and scorpions. For ancient sources, see Vitruvius.

[9] Of this opinion Russo, who assumes the distance of the battery of ballistae at about 100 metres from the city walls. He makes it clear that ballistae were flanked by other types of weapons with a curved shot, intended to strike inside the city walls (i.e. the onager).

precisely 86.3 m/s for balls with a radius of ca. 7 cm and with tense trajectories (Rossi, 2024, p. 12).

The possibility of documenting not only the craters in the surface of the walls, but also the projectiles that most likely caused these impacts, is a unique opportunity to compare both holes and projectiles on a dimensional level, and to define a measurement range within which the size of the impacting stone balls can be taken into account (as well as, of course, recording their constituent materials for virtual simulation purposes).

In Pompeii, today exhibited in the *Antiquarium* of Porta Marina (Fig. 13) and also preserved in the deposits of the excavations, there are several examples of stone projectiles attributed to the Sullan siege (F. Russo & Russo, 2005, p. 93).

According to the sources, stone balls of various sizes used for the ballistae were roughly spherical in shape. Some of these were found in numerous houses, especially in the north-western part of the city, where the most intense clashes took place. Scholars believe that the biggest, e.g. stone balls of a Greek talent (about 26 kg) (Di Marco & Molari, 2020), were thrown over the walls using onagers in parabolic trajectories.

With regard to the missiles, the archaeologist Van Buren mentions 86 objects "of the type of ballista ball" that were preserved in the 1930s in the small museum set up in the long brick building that stretches along the northern part of the western side of the Forum, undoubtedly a selection of the missiles that the engines of the Sullan army hurled at the walls of Pompeii together with those of the defenders (Van Buren, 1932, pp. 14–17).

Fig. 13: Picture of the stone balls on display in the *Antiquarium*. Credits: A. Rossi

From the pile of projectiles shown in the historic photograph (Fig. 14), collected from various parts of the excavated area without any documentation of the origin of the individual pieces, some stones that did not correspond to the type of projectile were eliminated according to the archaeologist's sorting: the projectiles should have been well rounded and not irregular stones, most of them in tufa, some in lava and sandstone, a few in limestone, none in marble.

The approximate diameters recorded for typical examples range from 10 to 23 cm,[10] with no inscription or intentional mark, but with damaged areas on their surface, as if this were the result of an impact (he notes several balls around the excavation site with lesions or signs of impact). In his opinion, other balls were found in the vicinity outside the Ercolano Gate, another actual area of bombardment, as shown in Plate V of Vol. I by (Piranesi, Piranesi, & Guattani, 1804), where it is possible to see some balls and fragments near the entrance, although no ball is mentioned in the reports of this excavation (Van Buren, 1932, p. 17).[11]

Nowadays the projectiles on public display in the *Antiquarium* are around fifty. For initial analysis, a case study of intermediate diameter size was selected for documentation through expeditive photogrammetry (Nikon D5200, 6000x4000 pixels, 35mm focal length lens), and for three-dimensional reconstruction (Fig. 15-a,b,c). Despite the uncooperative lighting and the stacked position of the balls in the museum, it was possible to reconstruct the 3D models of parts of

Fig. 14: Picture of the missiles in the Museum of the Forum. Year 1932 ca. Credits: Van Buren, 1932, Plate 2, I

[10] To be precise, he records 23, 21, 19, 16, 14, 13, 11 and 10 cm, without mentioning any conversion into Roman units of measurement.

[11] Van Buren mentions similar projectiles found at Pergamon in the late 1920s; two groups at Doura-Europos of various diameters found on the top of the walls in the 1930-31 campaign; many in excavations at Corinth in the early 1930s, as well as numerous projectiles of hard stone with diameters of about 10 cm found in Syria.

the objects (at the current stage of the research, the hidden parts are not fully documented and should be moved to a more favourable position for complete acquisition), then exported to Reverse Modelling software 3D Systems Geomagic Design X for dimensional and geometric analysis.

Some methodological aspects of reconstructing the geometry of the object, i.e. the possible steps to be taken to determine a reliable geometry as the centre and radius of the ideal sphere closest to the real object, should be considered:

1. Once the mesh has been uploaded into the software, the lower, less documented, and more uncertain part is eliminated by trimming one or more ranks of polygons belonging to closed sequences of edges defining holes. In facts, these polygons, being produced by software interpolation, deviate the most from the real object (Fig. 15-d);
2. The model is then automatically regionalised, with options that take into account the sensitivity depending on the roughness of the mesh (Fig. 15-e); this involves dividing it into areas highlighted in different colours, which are automatically recognised by the software and classified according to whether they are freeform or closer to solids (sphere, cylinder, etc.), which can be used to create surfaces or solids with basic primitives;
3. Using automatic primitives, it is possible to quickly extract simple solid geometric objects from a regionalised mesh; in this case, the shape to be automatically extracted is specifically requested as a sphere (partial or total NURBS surface or solid) (Fig. 15-f). The automatic surface extraction suggests a spherical surface, but it is still possible for the operator to modify the centre of the sphere, the vector passing through the centre and the semicircular revolution profile (Fig. 15-g). This proves to be useful, since the initial analyses have shown that the geometry of the possible projections can be assimilated to a non-perfect sphere; moreover, the search for several sections on the mesh gives measurements with a radius between 6.67 and 6.92 cm, that is to say with a distance of about 2.5 mm between different sections of the irregular mesh (Fig. 15-h). It is therefore necessary to define, for the virtual simulations, whether to consider a maximum or a minimum sphere in the given neighbourhood;
4. From the sectional polylines of the contour lines, it is also possible to obtain best-fitting circles

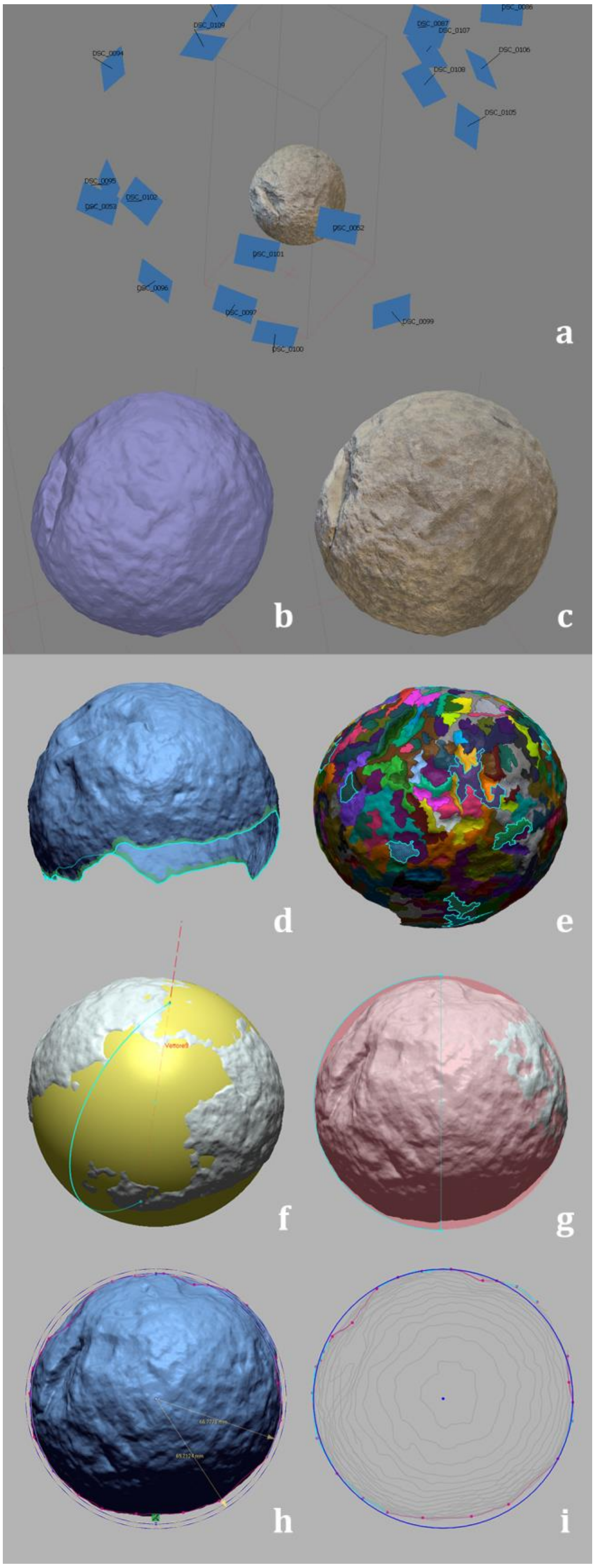

Fig. 15: 3D model of a stone ball: a) photogrammetric reconstruction; b-c) polygonal mesh w/o and with texture; d) edge trimming; e) regionalisation; f) automatic primitive extraction; g) radius of the inscribed/circumscribed sphere; h) best fitting circles from the 3D sections. Credits: S. Bertacchi

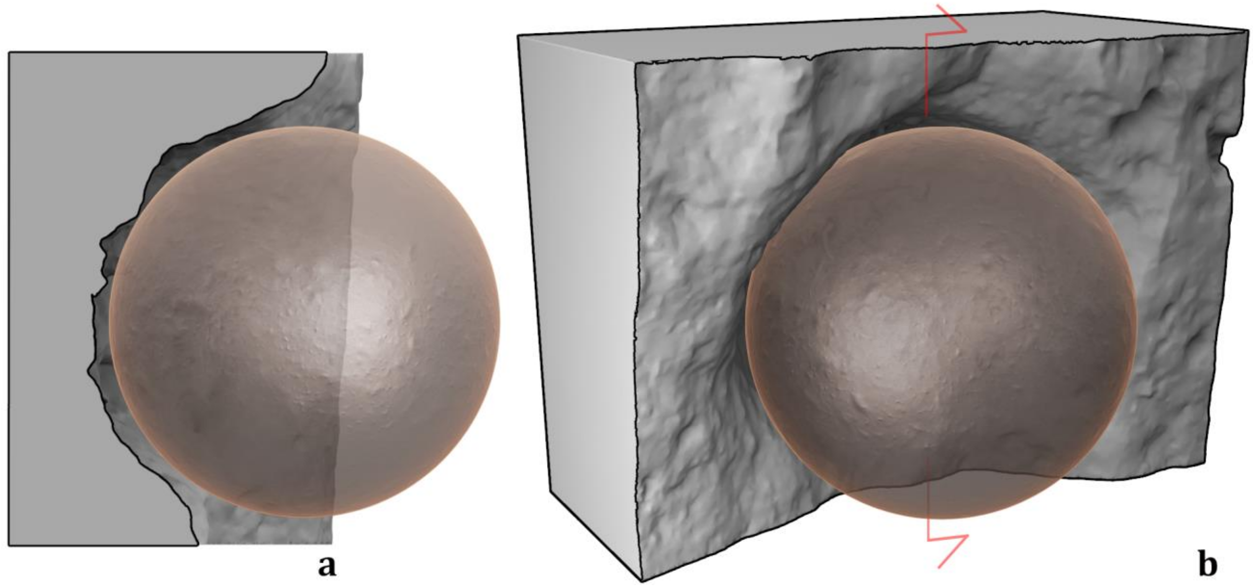

Fig. 16: Impact crater on the digital surface of the city wall and the reconstruction of a stone ball projectile consistent with the impact: a) vertical section view; b) perspective view. Credits: S. Bertacchi

by excluding segments belonging to hollow parts (in pink) due to lack of material (fall due to impact or stone defects during stone carving) (Fig. 15-i).

As a final consideration, once obtained reliable measurements , it may also be appropriate to compare them with ancient measurement unit to check for possible deviations, considering, for example, that the Roman palmus was equal to 1/4 *pes* (Roman foot) and thus measured approximately 7.41 centimetres (Molari et al., 2012). This would also correspond with the diameter of some of the cavities, particularly the one studied, shown in Fig. 10.

5. Conclusions

The photogrammetric technique was ideal for the morphometric documentation of the irregular stone surface, and the 3D model obtained corresponds to the resolution defined in the survey project as necessary for the subsequent restoration of the damage caused to the city walls.

Of course, for proper conservation it is necessary to be able to discriminate any anthropic traces present, which could be evidence of deliberate damaging actions carried out during the siege, precisely to avoid losing or obliterating them in case of restorations.

Digital means, in contrast to traditional contact techniques, allow a quick and accurate acquisition of geometric and radiometric information, and polygonal mesh software provide multidisciplinary teams with a set of tools that can enhance perception and visualise the model with different maps for research purposes, or even extrapolate dimensional information.

The subject, which is completely original, has these main results:

i) identification of impact marks scattered in large numbers on the northern section of the walls connecting the Vesuvio and Ercolano Gates, evidence preserved to this day of the siege by Sulla in the 1st century B.C., proposing a system for differentiating, with a certain degree of reliability, the anthropic traces produced by the impact of stone throwers (ballistae) hurled by the Roman artillery;

ii) assessment of the range of dimensions in which the projectiles can be included, based on a dimensional and geometric classification carried out on reality-based models. Reverse modelling techniques applied to digital replicas of the ballistic impacts made clearly legible and analysable in a virtual environment some important features, i.e. the diameter of the impact depressions, the penetration depth of the ballistic marks, and a description of the impact craters produced by the impacting projectiles;

iii) a detailed study of the shape and volume of the preserved stone spheres that were probably used as projectiles, essential to determine the mechanical properties of the masonry materials and to calculate the residual kinetic energy (Fig. 16);

iv) to provide experts with the necessary information to calculate the launching parameters, given the characteristics of the projectiles, their trajectory and the hardness of the stone ashlars, for the virtual and physical reconstruction of accurate and philologically functioning prototypes, in order to possibly rewrite or improve the history of the siege of Sulla and, by analogy, other sieges in which such Roman artillery was used;

v) in the wider context of experimental archaeology, also to provide a guideline with a correct workflow for the production of demonstrators for museum environments and to create a dedicated thematic platform to provide the appropriate support for the process of data production, preservation and management according to a systemic approach for a complete integration of knowledge, i.e. 3D galleries (on-line and off-line) to attract a wider audience of users, including non-experts or differently-abled, and to disseminate cultural content about the Imperial age.

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