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"LA REGOLA DELLI CINQUE ORDINI DI VIGNOLA". 3D PRINTED LEARNING MODELS

Fabrizio Avella, Daniela Butera*, Giulio Cellura**

*University of Palermo – Palermo, Italy.

Abstract

The study of architectural orders has been one of the cornerstones in the education of architects in Academies and Universities up until the 19th century. Although it has lost its original purpose of providing practical tools, the practice of teaching continued into the 20th century and remains a subject of study in school and university courses even today. The focus, often directed at understanding components and recognizing moldings, is still carried out today through redrawing exercises. To complement traditional methods, a teaching aid is being proposed, based on the composition of 3D printed models, designed in such a way that allows students to reassemble the elements of the order according to syntactic rules, helping them understand compositional principles. "La regola delli cinque Ordini di Vignola" (The Rule of the Five Orders by Vignola), one of the most influential treatises from the late Renaissance to the 20th century, was chosen as a reference.

Keywords

Vignola, Treatise of Architecture, learning 3D models, 3D printing.

1. *The classical order and the role of the treatise by Jacopo Barozzi da Vignola.*

Classical architecture cannot be understood without considering the syntactic structure of the order, which was developed in the archaic age as a stone transposition of the wooden temple, and then adapted in the late Hellenistic and Roman periods in the tectonics of arched and vaulted wall systems. The linguistic definition of the orders initially matured through an empirical process, where each construction provided an opportunity to perfect or reinterpret the characteristics of the order.

The first difficult attempt to codify these characteristics emerged at the end of the 1st century BC, when Vitruvius wrote *De Architectura* between 35 and 25 BC.[1] It is considered the first known architectural treatise and would become an essential reference for architectural theory and treatise writing from the Renaissance onwards.

The need for codification probably arose from the observation that, even in the mature Classical and Hellenistic periods, the architectural order was interpreted not only according to the prevailing 'taste' of the time and geographical area but could also vary in nearby, contemporary buildings.

It thus became necessary to establish the archetypal invariants of the order and how they could be interpreted without violating the "rules" which Vitruvius was able to define.

Before describing the various orders in detail, referencing the Greek models, Vitruvius establishes the necessary categories to define architecture, theorizing the famous triad: *utilitas*, *firmitas*, *venustas*, translated in the edition under examination as "comfort, strength, beauty"[2] (Galiani, 1790): "La fabbrica dunque […] poggia la sua bontà sopra tre fondamenti, e sono Comodo, Fortezza, e Bellezza"[3] (Galiani 1790, p. XIII).

The perfect balance of these three components of an architectural work is achieved through the categories described in Book I: *Ordinatio*, *Dispositio*, *Eurythmia*, *Symmetria*, *Decor*, and *Distributio*[4]. The first two are necessary to achieve functionality (*utilitas*) and respond not only to functional needs but are also part of a system of

[1] The precise dating remains uncertain up to now.

[2] In this context, reference is made to the one translated by Bernardo Galiani, Architect of Merit of the Accademia of S. Luca, published in 1790, which present a critical introduction considered useful tu explain the theory of Vitruvio.

[3] The passage is included in the introductory chapter *Idea Generale dell'architettura Estratta da' Dieci Libri di M. Vitruvio Pollione*.

[4] Book I, *Definizione e parti dell'Architettura. Scelta del sito per gli Edifici pubblici*, Chapter II, *Di che si formi l'Architettura.*

rules underlying the construction of the entire building.

"L'Ordinazione è un misurato comodo de' membri di una fabbrica presi separatamente, ed il rapporto di tutte le sue proporzioni alla Simmetria: si regola quella dalla Quantità […]: la Quantità poi e la giusta distribuzione dei Moduli presi dalla stessa opera, e adattata a ogni membro di ciascuna parte della medesima" (Galiani, 1790, p. 7-8). The "members of building" must be dimensioned according to functional measures that respect the proportions derived from the Module, a reference measure necessary to achieve Symmetry through proportions.

"La Disposizione è una propria situazione delle cose, ed un vago effetto dell'opera negli accordi per cagion della Qualità. Le specie della Disposizione, le quali in greco si chiamano Idee, sono la Pianta, l'Alzato, e la Prospettiva" (Galiani, 1790, p. 8). By "disposition" we should understand the correct positioning of the elements: borrowing from Renato De Fusco's commentary, "sembrerebbe che disposizione sia un termine inclusivo del precedente ordinatio […] la differenza sembra consistere nel fatto che dispositio sia la proprietà di collocare al posto giusto o delle parti di una fabbrica in ordine alla sua qualità", controllandola per mezzo dell'ausilio dei metodi di rappresentazione della doppia proiezione ortogonale e della "prospettiva" (De Fusco 1968, p. 20)[5] . Architectural elements, sized according to functional criteria and proportioned based on the module, must be arranged according to the correct syntax.

Venustas is achieved through the careful composition of parts according to the rules of *Eurythmia*, *Symmetria*, and *Decor*.

"L'Euritmia è il bello, e grato aspetto cagionato dalla disposizione delle membra. Si ha, quando di dette membra corrisponde l'altezza con la larghezza, e la larghezza con la lunghezza, ed in somma tutte le cose hanno la loro giusta proporzione" (Galiani, 1790, p. 9). The definition has been subject to various interpretations and may seem repetitive, but the commentary that Galiani himself includes in footnote may prove useful: "L' Euritmia è quella, che insegna l'eguale distribuzione dei membri d'un edifizio, acciocché facciano grato aspetto" (Galiani 1790, p. 9). It is not enough for the elements to be correctly sized, respecting a proper Ordinatio, or that they are placed according to a correct Dispositio; they must also be distributed in a way that results pleasing to the eyesight.

Symmetria is the category that has sparked the most historiographical debate regarding possible interpretations. The text of Vitruvius states: "La Simmetria è un accordo uniforme fra le membra della stessa opera, ed una corrispondenza di ciascuna delle medesime separatamente a tutta l'opera intera: sìccome nel corpo umano vi è Simmetria fra il braccio, il piede, il palmo, il dito, e le altre parti, così lo stesso è anche in ogni opera perfetta. E primieramente nei tempi si cava il Modulo dalla grossezza delle colonne, o dal Triglifo […]; così in tutte le altre opere da qualche membro si cava la misura della Simmetria" (Galiani, 1790, p. 9). It seems, therefore, that *Symmetria* should be understood as the "principio fondamentale della perfezione estetica" (Panofsky, 1962, p. 91) achieved as a result of the theory of proportions understood as "fondamento razionale della bellezza" (Panofsky, 1962, p. 91). About aspects related to anthropometric and musical proportions, reference is made to specific studies[6] (Panofsky 1962, Wittkover 1964, Rossi 1996).

The third component necessary to achieve what Vitruvio call *venustas* is "Decoro": "Il Decoro è un raffinato aspetto dell'opera, composto di cose approvate dalla ragione: questo si regola o dallo Statuto, […], dalla Consuetudine, o dalla Natura" (Galiani, 1790, p. 9).

The Statuto mandates the correct attribution of buildings: "A Minerva, a Marte, e ad Ercole sì faranno edificj dorici; imperciocché a quelli Dei convengono a cagion del loro valore edificj senza delicatezza. A Venere, a Flora, a Proserpina, ed alle Ninfe dei forti li faranno proprj edificj Corinzj, perchè riflettendoli alla gentilezza di quelli Dei, parrà che i lavori delicati ed ornati di fiori, frondi, e volute accrescano il proprio loro decoro. A Giunone, a Diana, a Bacco, e ad altri Dei di tal somiglianza si terrà la via di mezzo, facendo gli edifici Jonici, i quali faranno proprj, perché partecipano della sodezza Dorica, e della delicatezza Corintia" (Galiani, 1790, p. 10).

[5] The definition of the term *perspective* is not addressed in detail, as it has sparked a long debate about what Vitruvius truly meant and whether he was referring to a projection system similar to the perspective codified during the Renaissance.

[6] The bibliography on this topic is extensive. We limit ourselves to suggesting the reading of Panofsky's essay, "*La storia della teoria delle proporzioni del corpo umano come riflesso della storia degli stili*".

"Consuetudine" determines the congruence of the various parts of a building: for example, "Sarà Decoro di Consuetudine, quando ad edificj magnifici nell' interno si adatteranno anche entrate proporzionate, e magnifiche" (Galiani, 1790, p. 10); or: "Così parimente sé nei corniciami Dorici si scopriranno dentelli nella cornice: o sopra capitelli, e colonne Joniche s'intagliassero Triglifi nelle cornici, trasportando così le cose proprie d'un Ordine in un altro, lì offenderà la vista, poiché sono già state nei tempi addietro stabilite consuetudini diverse e proprie in ciascun Ordine" (Galiani, 1790, p. 10).

Natural "Decoro", finally, establishes the rules regarding the healthiness of a building, the choice of a site based on its characteristics, and the correct positioning of spaces in relation to the sun to achieve lighting that is appropriate for their function: "Il Decoro Naturale poi sarà quello: primo sé per ogni tempio si sceglieranno siti di buona aria, con fonti d'acqua sufficienti, ed ivi si fabbricheranno. […] Parimente Decoro Naturale sarà, sé nelle camere, e nelle librerie si prenderanno i lumi dall'Oriente: nei bagni, e nelle danze d'inverno dall'Occidente jemale: nelle gallerie, ed ove si richiede un lume sempre eguale dal Settentrione […]" (Galiani, 1790, p. 10).

Regarding firmitas, Vitruvius summarizes the necessary characteristics in "Distributio", which encompasses both constructive aspects, primarily the choice of materials, and those related to the rationalization of the building based on function and expenditure: "La Distribuzione è il comodo uso del materiale, e la parca spesa nei lavori moderata dalla ragione" (Galiani, 1790, p. 11).

It has seemed necessary to provide a summary of the Vitruvian categories because *De Architectura* becomes one of the foundations of architectural theories during the Renaissance and in the following centuries. The first edition that seems to have played a significant role in its dissemination was published in Rome by Sulpicio da Veroli between 1486 and 1487, followed by the Florentine edition of 1496 and the Venetian edition of 1497. Other notable editions include the 1511 publication in Venice (reissued in 1513, 1522, and 1523) edited by Fra' Giocondo, the famous edition edited by Cesare Cesariano, published in Como in 1521, and the equally renowned translation and commentary by Daniele Barbaro, accompanied by drawings by Palladio and published in Venice in 1556. The editorial success of the Vitruvian treatise would continue until the 19th century, but in this context, we focus on highlighting its importance within the Renaissance.

Editions, commentaries, and interpretations of Vitruvius's work stimulate the writing of treatises, and even in cases where the approach is critical or redefines the proportional systems of the orders, it is still regarded as a methodological reference: "Sulla scia delle indicazioni di Vitruvio, sottoposte in tempi diversi a faticose esegesi, Alberti, Serlio, Vignola, Palladio, Scamozzi e tanti altri, hanno tentato di definire la configurazione sino agli aspetti più minuti e di strutturare i rapporti tra le misure delle diverse parti degli ordini, allo scopo di costruire un sistema condivisibile […]" (Dotto, 2022, p. 55).

The treatise by Vitruvius is also referenced by Jacopo Barozzi da Vignola (Vignola 1507 – Rome 1573), and although he proposes different proportional systems in some cases, there is no doubt that the aforementioned categories can be found in his codification. The fact that Vitruvius is a reference is confirmed by the dedication. "Ai lettori" (to the readers) present in the preface: "[…] se qualchuno giudicasse questa fatica vana con dire che non si può dare fermezza alcuna di regola, attesoché secondo il parere de tutti, et massime di Vitruvio molte volte conviene crescere ò scemare delle proportioni de membri dell'ornamenti per supplire con l'arte dove la vista nostra per qualche accidente venghi ingannata; a questo gli rispondo; in questo caso essere in ogni modo necessario sapere quanto si vuole che appaia all'occhio nostro, il che sarà sempre la regola ferma che altri si hauera proposta di osservare […]"[7] (Vignola, 1562, p. III).

A determining figure in the Renaissance landscape, both as an architect (Affanni & Portoghesi, 2011) and as a theorist and treatise writer, he would become a reference point in both the Italian and international arenas. The first edition of *La regola delli cinque ordini d'architettura*, published in Rome in 1562, emerges after more than a century during which treatises have established themselves as a fundamental vehicle for the formulation and dissemination of architectural theories. Referring only to the Italian scene, the following have already been published: *De re aedificatoria libri*

[7] Il brano conclude la dedica "All'Ill.mo S.re mio et Signore Singulariss.o il Car. le Farnese" (Vignola 1562, p. III).

decem by Leon Battista Alberti (Rome 1452), the *Trattato di Architettura* by Filarete (Florence 1460), and the *Trattato di architettura civile e militare* by Francesco di Giorgio Martini. (completed at the court of Urbino after 1482, the year of the death of Federico di Montefeltro), some of the books from Serlio's *Trattato di Architettura*, published between Venice and Paris (Book IV in 1537 in Venice, Book III in 1540 in Venice, Books I and II in Paris in 1545, Book V in Paris in 1547 and in Venice in 1551, the Libro estraordinario in Lyon in 1551 and in Venice in 1557, while Book VII will be published in Frankfurt in 1575 and in Venice in 1584), and the *Quattro libri* di Palladio, published in Venice in 1570, but already drafted in 1556. Following these, other treatises that have shaped architectural theory, both in Italy and abroad, will emerge. In this landscape, which is impossible to summarize in this context, Vignola's treatise stands out as one of those that had the greatest dissemination, with hundreds of editions published until the end of the nineteenth century (Thones, 2002, p. 333-377).

The attention to the treatise becomes evident immediately after its first publication, and it is emblematic that it becomes part of the training of architects at the Accademia di San Luca: "Si studiano i cinque ordini del Vignola da cui si apprende quale sia la differenza, che corre dall'uno all'altro, quali siano le misure dei piedistalli, delle colonne, e dei cornicioni di ciascun ordine, i loro modini, secondo le antiche fabbriche più regolate, e più perfette, si impara poi a ricopiargli bene, e pulitamente toccargli d'acquerello, e per maggiormente impratichirsi, si ricopiano ancora delle porte, e delle finestre di qualche accreditato professore, e quelle inventate dal suo maestro, o quelle di qualche altro architetto moderno, e vivente, che la voce del popolo abbia molto applaudite. Poi si passa a far qualcosa di sua invenzione." (Bottari 1754, p. 110-111, included in Manfredi 2008, p. 30-31.). The study of Vignola's treatise thus becomes part of the complex educational process for architecture students, who master the rules, study the solutions of contemporary masters, and only then can engage in their own compositions.

The *"Regola delli cinque ordini"* acquire a crucial role in defining architectural language, not only in the late Renaissance but also in the following centuries, across both the Italian and European landscapes: Della Porta, Maderno, Carlo and Domenico Fontana, Bernini, Fontana, Fuga, Juvarra, Vanvitelli, Vittone, Marvuglia in Italy; Lemercier, Perrault, Mansart in France; Fischer von Erlach in Austria; Johann Dientzenhofer in Germany; Christoph and Kilian Ignaz Dientzenhofer in the Czech Republic; De Toledo, De Herrera, Cano in Spain; Álvarez, Ludovice, Dos Santos in Portugal are just a few of the many architects, by no means not an exhaustive list, who applied these principles in various cultural and geographical contexts.

The reasons for the success of the treatise are manifold. Vignola does not address an erudite audience nor dwell on speculative discussions; instead, he conveys the explanation of the "regole" through a detailed iconographic apparatus, described in essential and concise texts. He does not tackle the topic of architectural typologies, which is present in many other treatises, nor does he differentiate between applications in religious, civil, or military contexts. Vignola also does not adhere to a specific metric system, allowing the "regole" to be applied using any unit of measurement, making them adaptable to any geographical area. He describes the orders with great clarity, providing proportional guidelines for the sizing of architectural elements, leaving architects the freedom to adapt them according to various needs. The drawing of the orders does not require complex graphic operations (Migliari, 1991). "[…] il metodo di Vignola propone uno schema che permette di determinare ciascuna parte dell'ordine desumendola direttamente dalle misure generali" (Dotto, 2022, p. 63), with simple arithmetic operations of proportional calculation (Carpo, 2003).

It becomes a highly refined instruction manual in which the grammatical rules are so clear that they can be applied in countless syntactical variations: "La semplicità applicativa del metodo di Vignola è la caratteristica che ne ha decretato probabilmente lo straordinario successo che in modo crescente ha visto la Regola come uno dei lavori più influenti sino alle soglie del XX secolo." (Dotto, 2022, p. 63).

2. The plates in the treatise

The first edition of the treatise is illustrated with 32 plates, in which the five orders, Tuscan, Doric, Ionic, Corinthian, and Composite, are described, along with drawings that depict a spiral column, portals, and windows. For each order, the overall dimensions of the colonnade and the arched wall structures, either resting on the

ground or on a pedestal, are presented, along with the details of the pedestal itself, the top of the column with capital details, the frieze, and the cornice. The Composite order is an exception, as no overall drawings are included because its general dimensions share the same proportions as the Corinthian order.

Some notes: the columns of the Doric, Ionic, and Corinthian orders are shown without fluting in the overall drawings, while they are fluted in the detailed drawings. The capital of the Doric order is shown with both a smooth echinus and one with egg-and-dart decoration. In the detailed plates of the Ionic capital, the complex construction of the volute is illustrated (Fig. 1).

The drawings, starting from the first version, are all enriched with stippling in certain areas and shaded with single lines, allowing for lithographic reproduction. In later versions, additional plates are included, sometimes reaching up to 37, and a synoptic plate is introduced, where the individual columns of the five orders with their entablature are displayed side by side, highlighting the different proportional dimensions.

All editions faithfully reproduce the proportional systems of the first edition, both in the overall dimensions and in the detailed ones.

There is also an iconographic fidelity that is surprising: almost always, the images are reproduced very faithfully to the original drawings, and only in a few cases are there variations that do not alter the proportions.

The drawings clearly show the system based on the module, with multiples and submultiples necessary for sizing architectural elements, but do not refer, as already mentioned, to a precise unit of measurement. This allows the system to be applied with any unit of measurement, being based on an essentially numerical breakdown. It therefore seemed reasonable to use the decimal metric system, taking the centimeter as the unit of measure for the module. The models, in this way, can be easily measured during assembly or after assembly is complete. Indeed, while there may be some difficulty measuring moldings that have submultiples with a denominator that is a multiple of 3, it is extremely easy to measure the dimensions of the main elements and the spacings.

3. Criteria for the creation of models

The study of the orders has remained a part of an architect's education to this day, and often, as was the case in the past, redrawing is used as a tool for deepening understanding and memorizing shapes, stylistic elements, and nomenclature. Frequently, teaching in schools, universities, and academies has utilized wooden or plaster models as references for drawing from life: "Gli allievi dovranno disegnare assiduamente da questi modelli, poiché questo li aiuta a sviluppare non solo una giusta visuale e un'adeguata valutazione della forma, ma anche la conoscenza delle luci e delle ombre, dei diversi atteggiamenti e dei diversi scorci."[8] (Sulzer, 1792, p.12).

The model, therefore, serves not only as an object for redrawing but also to observe a piece of architecture in its three-dimensional reproduction, capturing volumetric relationships and interaction with light (Tamberino et al., 2017).

The idea of disassemblable three-dimensional models arises from the aim of providing students not just with a three-dimensional object that highlights its morphological characteristics and visualizes the effect of light, but also with a system that, through the process of assembling elements, helps them understand that the architectural order cannot be seen merely in its stylistic or linguistic definition. Instead, it is the result of a complex logical and compositional system dictated by precise rules and must be framed within an overall semantic structure.

Thus, it was decided to develop models that could represent the decomposition into elements that, combined according to the rules of the order, acquire a precise meaning (Bianconi & Filippucci & Magi Meconi, 2018)[9]. The purpose is purely educational, with specific goals: to allow students to assemble the pieces until they achieve the correct result and to enable the teacher to explain the Vitruvian categories also through intentionally incorrect reconstructions. For example, if a column is assembled with the capital placed where the base should be, the result will yield individual components that respect their correct dimensions according to the right quantity dictated by the "Ordinazione", but do not conform to the proper positioning of a correct "Disposizione".

[8] The translation from Pinelli is provided in 1985, p. 110 of the passage by J.G. Sulzer.

[9] The logic of decomposing elements in the modeling of Vignola's orders has already been the subject of study. In that case, the goal was the parameterization of the elements of the order.

Fig. 1: Synoptic table relating to the orders, arranged from left to right: the architectural order, the trilithic system, the arch system.

Alternatively, if, after assembling the Doric order pieces in the correct sequence of pedestal, column, and entablature, an Ionic entablature is placed on top, the Decorum of Custom will be compromised. Furthermore, if in the trilithic system the positioning of the column does not respect the measurement of the intercolumn, the system will be incorrect as there is neither a proper Arrangement nor the necessary Eurhythmia to achieve Symmetry.

The design of the system has considered, from the beginning, a fundamental aspect: if the architectural order constitutes a language, it is possible to identify grammatical elements that, composed according to shared rules, can give rise to phrases, sentences, and narratives that underlie the various syntactic interpretations of individual architects. This is why, even during the phase of developing the three-dimensional models, a non-random decomposition was designed to respect the grammatical structure of the architectural order. For example, the trilithic system was discretized into column and entablature, or the column was decomposed into base, shaft, and entablature. It is necessary to make a clarification: the models do not reproduce the construction system but respect the linguistic structure; in some cases, the two systems coincide, but in others, they do not necessarily, as in the case of the arch structure, where the adopted construction solutions do not necessarily realize the wall portion and the semi-column or three-quarters of a column with separate architectural elements. (Paragraphs 1-2-3 written by Fabrizio Avella).

4. Decoding and digital representation of Vignola's architectural orders

"Dovendoli trattare delli cinque ordini di Colonne, cioè Toscano, Dorico, Jonico, Corinto e Composito, si pongono tutte in principio con i loro piedistalli, e con le loro cornici, per dare una notizia generale di quanto a parte a parte minutamente sarà descritto.". With these words, Vignola provides an introductory overview with the description of the architectural orders, which will be expanded upon in subsequent plates with detailed drawings. In fact, he continues: "Il modulo, che serve all'ordine Toscano, ed il Dorico, si divide in 12 parti, e quello, che si vuol formare per l'ordine Jonico, o per il Corinto, ovvero per il Composito, essendo questi tre ordini composti di membri più gentili, si dovrà dividere in parti 18; come a suo luogo si dirà. Già è noto a tutti, che il detto modulo è una misura adattabile a tutte le grandezze" (Vignola, 1736, p. 7).

Starting from these considerations, the process of redrawing and modeling the orders was carried out, using the modules and parts described by Vignola in his treatise as units of measurement. Through using Rhinoceros drawing and modeling software, it was possible first to perform a 2D redrawing operation and then a 3D modeling (Lo Turco, 2012). This allowed for a faithful digital translation while also ensuring an accurate geometric reconstruction of the present moldings.

4.1 The Tuscan order

The first step was to redraw the Tuscan order based on the plates from Vignola's treatise, respecting the geometric constructions underlying Vignola's representations. A large portion of these constructions are already present in the Tuscan order and recur in the subsequent orders.

In this regard, Vignola introduces a detailed nomenclature in the Tuscan order, which is not repeated in the plates of the later orders but is expanded only by the unique moldings of each order. Below are the moldings for which it was necessary to follow strict geometric principles, essential for their correct reconstruction:

Cyma reversa (Gola Rovescia): To draw the cyma reversa, the following steps were taken: with centers at points A and O and radius OA, two arcs were drawn that intersect at point C. The same procedure was repeated for OB, yielding point D. With center at C and radius CA, the arc OA was drawn. This procedure was repeated with center at D and radius BO. The cyma reversa thus obtained was then transformed into 3D using a one-rail sweep (Fig. 2).

Torus (Toro): To draw the torus, with center at O, a semicircular arc was traced. The resulting semicircle was then transformed into 3D through a revolution operation (Fig. 3).

Astragal (Tondino): The astragal was drawn similarly to the torus, both in its 2D geometric construction and its 3D modeling.

Ovolo: To draw the ovolo, with center at O, a quarter-circle arc was traced. As with the previous examples, the 3D version was obtained using a revolution operation (Fig. 4).

The result is the complete 3D model of the Tuscan order (Fig. 9). This model was verified against the module proportions described by Vignola, who states: "Ma dovendosi fare

quest'ordine Toscano col suo piedistallo, si partirà tutta l'altezza in parti 22 ⅙, e con ciò sia, perché il piedistallo ricerca avere in altezza la terza parte della sua colonna con la base, e capitello, che essendo moduli 14, la terza parte sono moduli 4 ⅔, i quali aggiunti a 17 ½, vanno al numero di 22 ⅙." (*Ivi*, p. 13).

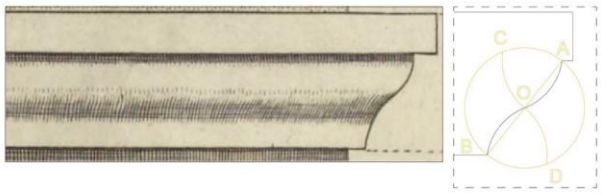

Fig. 2: Geometric construction of the Cyma Reversa.

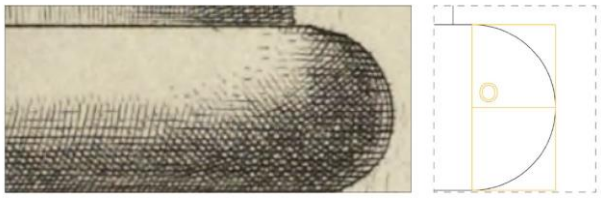

Fig. 3: Geometric construction of the Torus.

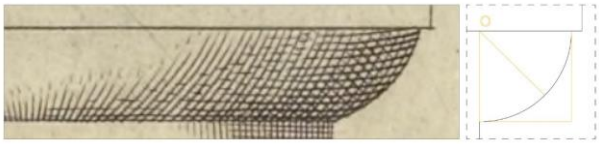

Fig. 4: Geometric construction of the Ovolo.

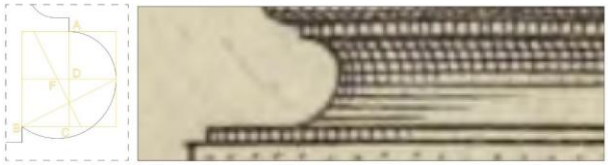

Fig. 5: Geometric construction of the Scotia.

4.2 The Doric order

What was previously described has been applied in the reconstruction of the Doric Order, as it too is composed of moldings whose geometric constructions reflect elements found in the Tuscan Order. In this case as well, the model was verified for compliance with the modules described by Vignola (Fig. 10): "Il modo di fare la divisione di quest' ordine Dorico senza il piedestallo si è, che partita tutta la sua altezza in parti 20, di una di queste parti se ne fa il suo modulo, il quale pur li divide in parti 12, come quello del Toscano. Alla base, coll'imoscapo della colonna, si darà un modulo. Il fusto della colonna, senza l'imoscapo, si farà di moduli 14. Il capitello sarà un modulo. L'ornamento poi, cioè architrave, fregio, e cornice, faranno moduli 4, che è la quarta parte della colonna compresovi la base, ed il capitello, come si è detto addietro dover essere l'architrave M. 1, il fregio M. 1 ½, e la cornice M. 1 ½, che raccolti questi insieme sono M. 4, ed aggiunti agli altri fanno 20." (*Ivi*, p. 19).
"Il piedistallo Dorico deve essere moduli 5 ⅓ in altezza." (*Ivi*, p. 25).

4.3 The Ionic order

The Ionic order features a molding not found in the previous orders, namely the Scotia or upper cavetto. This is obtained in the following way: given points A and B, referred to a horizon, they are to be connected with a scotia. A horizontal line passing through B is drawn. From A, a vertical line is drawn, meeting the horizontal line at point C. The axis of segment AC is drawn, identifying the midpoint D. With center at D and radius AD, an arc is drawn that intersects the axis of AC at point E. Segment BE is then drawn. The axis of BE intersects the axis of AC at point F. With center at F and radius EF, the arc EB is drawn, completing the figure. The resulting geometry has been transformed into a 3D element through a revolution operation (Fig. 5).

Particular attention was paid to the redrawing of the volute (Salviati, 1552 & Dotto, 2016), as described by Vignola: "Tirato il cateto di questa prima voluta et un'altra linea in squadro che passi per il centro dell'occhio si divide il detto occhio […], et si comincia poi al primo punto segnato 1, et si gira col compasso una quarta di circolo dipoi al punto segnato 2, si gira l'altra quarta et così procedendo si fa i tre giri compitamente. Per far poi la grossezza del listello sì come egli è la quarta parte della larghezza che lascia di sopra il primo giro così si ha da partire ciascuna di quelle parti c'hanno servito per centri in 4 et girando poi oltre, 12 quarte di circolo con quelli centri sarà fornita." (Vignola, 1562, p. XX).

After drawing the spiral, the three-dimensional surface of the volute was obtained through the rotation and translation of the profile section (Fig. 6).

The overall proportions of the Ionic Order model is consistent with the modular system described by Vignola (Fig. 11): "Dovendosi fare l'ordine Jonico senza il piedestallo, il quale va diviso in parti 18, e questo avviene, che per essere

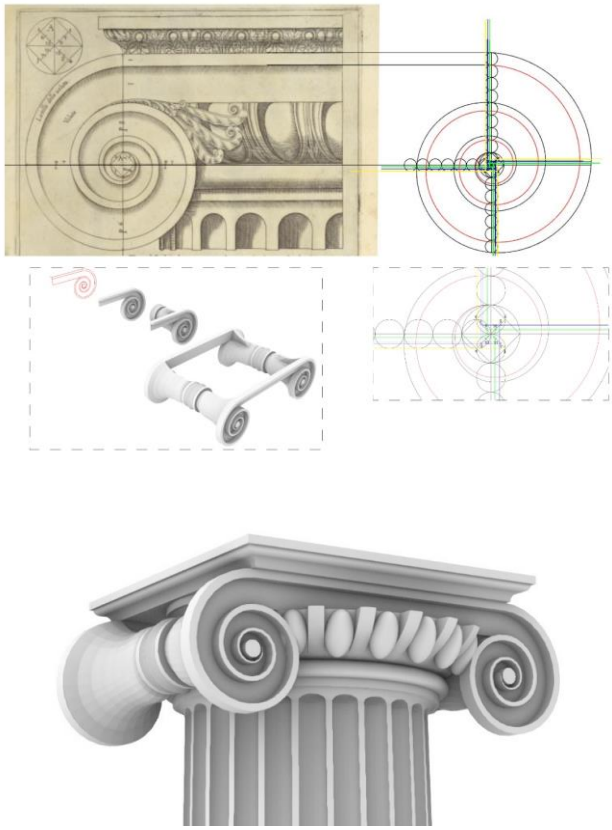

Fig. 6: Geometric construction of the spiral and 3D modeling of the volute. Below is a detailed view of the Ionic capital, complete with all its components.

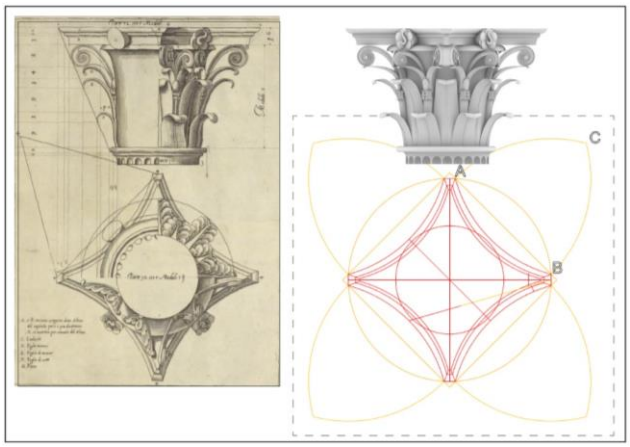

Fig. 7: Geometric construction of the abacus and front view of the 3D model of the Corinthian capital.

ordine più gentile del Toscano, e del Dorico, ricerca più minuta divisione. La sua Colonna deve essere 18 moduli, compresovi la base, ed il capitello; l'architrave moduli 1 ¼, il fregio moduli 1 ½, e la cornice moduli 1 ¾. che colti insieme architrave, fregio, e cornice Sono moduli 4 ½, che è la quarta parte dell'altezza della Colonna." (Vignola, 1736, p. 31).

"Ma dovendosi fare portici, o logge di ordine Jonico con li piedestalli, tutta l'altezza va partita in parti 28 ½, essendo il piedestallo con il suo ornamento moduli 6" (*Ivi*, p. 35).

4.4 The Corinthian order

The Corinthian order does not feature moldings with geometric constructions that are not already present and described in the previous orders. One of its peculiarities is its capital, which consists of two structural parts and a rich decorative apparatus. The structural components are the bell and the abacus, while the decorative elements include the lower leaves (*foglie di sotto*), middle leaves (*foglie di mezzo*), minor leaves (*foglie minori*), *caulicoli*, and flowers (*fiori*). The lower and middle leaves, arranged on two levels, are eight for each. The smaller leaves, also eight, are located between the middle leaves and have the shape of a chalice, from which the caulicoli emerge. The abacus has a square shape, with symmetries repeated throughout the capital, which is composed of four symmetric parts.

The redesign of the abacus, in particular, required a rigorous geometric construction, as it dictates the symmetries of all the other previously described elements. To draw the abacus, one starts with a square that has sides measuring three modules: with the center at the intersection of the two diagonals, a circle is drawn with a radius of two modules. This circle intersects the sides of the square at eight points. Each pair of these points provides the thickness for the four straight sides of the abacus. To draw the concave profile, an equilateral triangle is constructed with a side equal to the net distance between the vertices A and B, and two arcs are drawn with centers at A and B, intersecting at point C. With center at C, the arc of the circumference AB is traced.

This procedure is repeated for all four sides of the square, thus obtaining the geometry of the abacus. (Fig. 7).

The model of the Corinthian Order has proven to be consistent with what is described by Vignola (Fig. 12): "Per fare quest' ordine Corinto senza piedestallo: tutta l'altezza fi divide in parti 25, e con una di queste fi fa il modulo, il quale poi fi divide in parti 18, come quello del Jonico.

Le altre divisioni principali si veggono, e la larghezza da una colonna all' altra deve essere

moduli 4 ⅔, sì perchè gli architravi di sopra non patischino, come anche per accordare, che i modiglioni della cornice, nel suo eguale Spartimento venghino sopra il mezzo delle colonne." (*Ivi*, p. 45).

"Ma se si avranno a far logge, ovvero portici co' piedestalli, si partirà il tutto dell'altezza in parti 32, e di una di quelle parti si farà il modulo, 12 delle medesime farà la larghezza del vano, e 25 l'altezza" (*Ivi*, p. 49).

4.5 L' ordine Composito

The redesign of the Composite Order did not involve the geometric study of the elements that compose it. Instead, it was realized through the reuse of the geometries previously described. An example is the structure of the capital, which is derived from the Corinthian capital, as well as the volutes, derived from the Ionic capital (Fig. 8).

In this case as well, the model of the Composite Order has proven to be consistent with what is described by Vignola (Fig. 12): "Questo piedestallo Compofito tiene le medesime pro- porzioni del Corinto: folo è variato ne' membri della cimaſa, e baſſamento, come ſi può conoscere." (*Ivi*, p. 57). "Questa pianta, e profilo del capitello Composito tiene le medesime proporzioni del Corinto" (*Ivi*, p. 59) (Paragraph 4 written by Giulio Cellura).

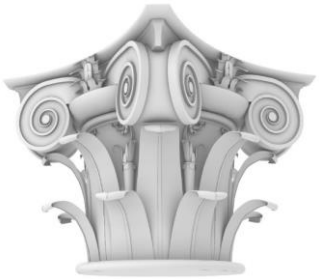

Fig. 8: Detail view of the Composite capital.

5. Preparation of the model and 3D printing

The process of preparing models for printing is essential for achieving a successful outcome. The procedure consists, in the first phase, of designing the model based on the considerations outlined in the previous chapter, which are preparatory for the operations of 2D and 3D digital redrawing.
Subsequently, it is necessary to formulate considerations regarding the scaling factors since Vignola, as previously mentioned, designs and represents the five orders using a modular system. Therefore, for practical reasons, a scaling factor was chosen where one module equals one centimeter on the printing plate.
Next, it was necessary to decompose the models into their individual elements and compositional sub-elements (Fig. 13), adapting to the type of printer used, the Bambulab X1C, which has a print plate size of 24x24x24 cm. The specifics of each model were adapted to the 3D printing criteria, which, in this case, cannot achieve a thickness less than 0.2 mm; consequently, to ensure visibility of every element, even the smallest details were conformed to this dimension to ensure good printing results.
In the next step, the model was exported to an STL file format, which stands for "STereoLithography." This format contains information about the geometric surface of a three-dimensional object, mediating between the 3D model information and the 3D printing data.
The process involves melting plastic filament through a heated extruder and positioning it with an overlap in layers according to the data provided by the printer. As the various layers are deposited, they fuse together, creating the physical object. This procedure was thus adopted for the three systems conceived by Vignola, namely: the trilitic system, the arch system, and the column system; each repeated for all five orders addressed by Vignola.

5.1 Printing Parameters

3D printing parameters must be adapted to the printer used, adjusting the settings based on the printing material and the desired level of detail. In this case, the parameters were set using slicing software, OrcaSlicer.

Specifically, the following were configured: nozzle diameter, layer height, number of perimeters, infill percentage, and material information.

The nozzle diameter determines the filament output size and influences the line width and details; the layer height describes the thickness of each printed layer, affecting both resolution and printing time; the number of perimeters establishes the number of material layers printed around the outer perimeter of each level of the model; the infill percentage defines the amount of material used to fill the interior of the model, determining its strength and weight.

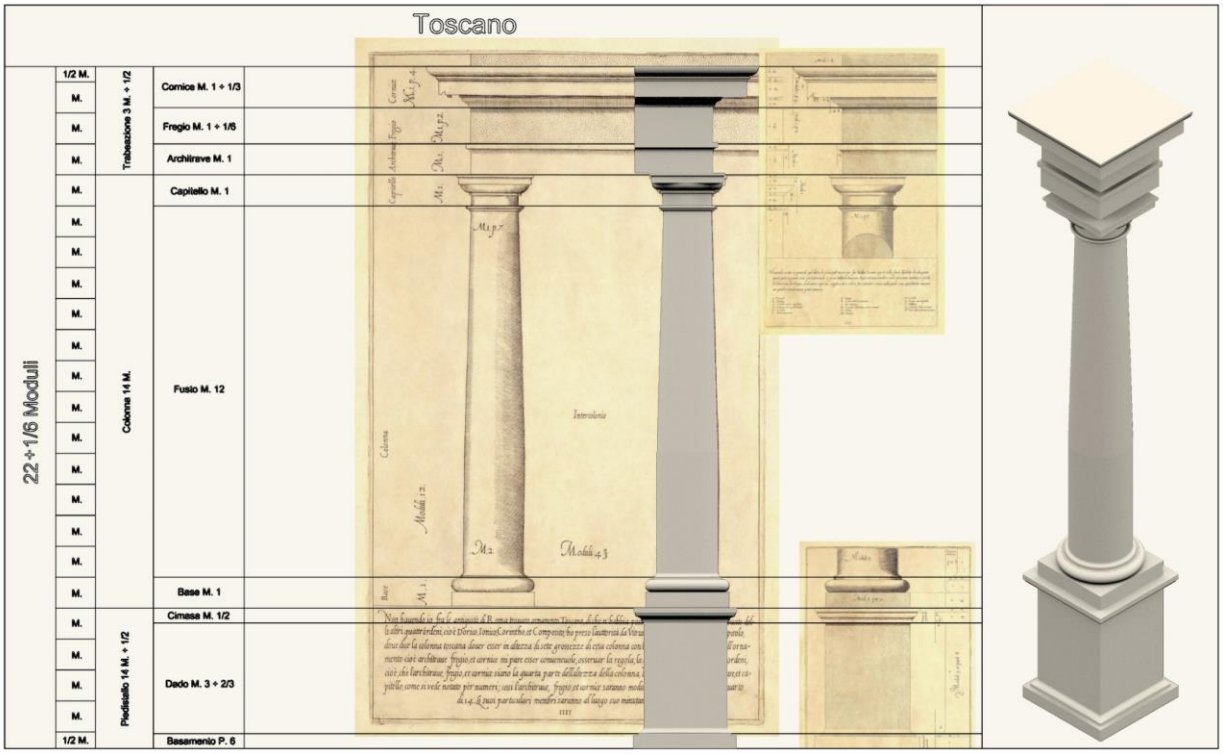

Fig. 10: Front and axonometric view of the 3D model of the Tuscan order, superimposed on the tables of the 1562 treatise. The modular scheme highlights the respect for proportions.

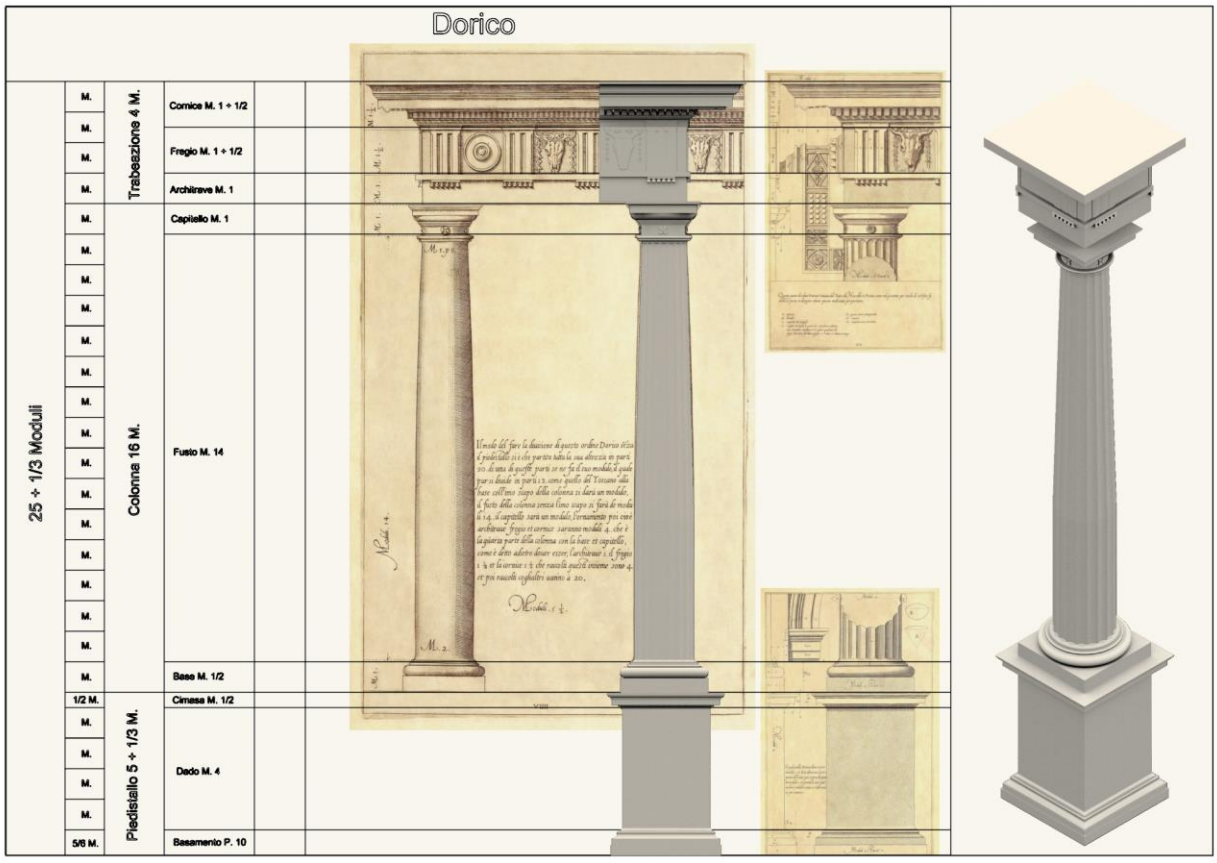

Fig. 9: Front and axonometric view of the 3D model of the Doric order

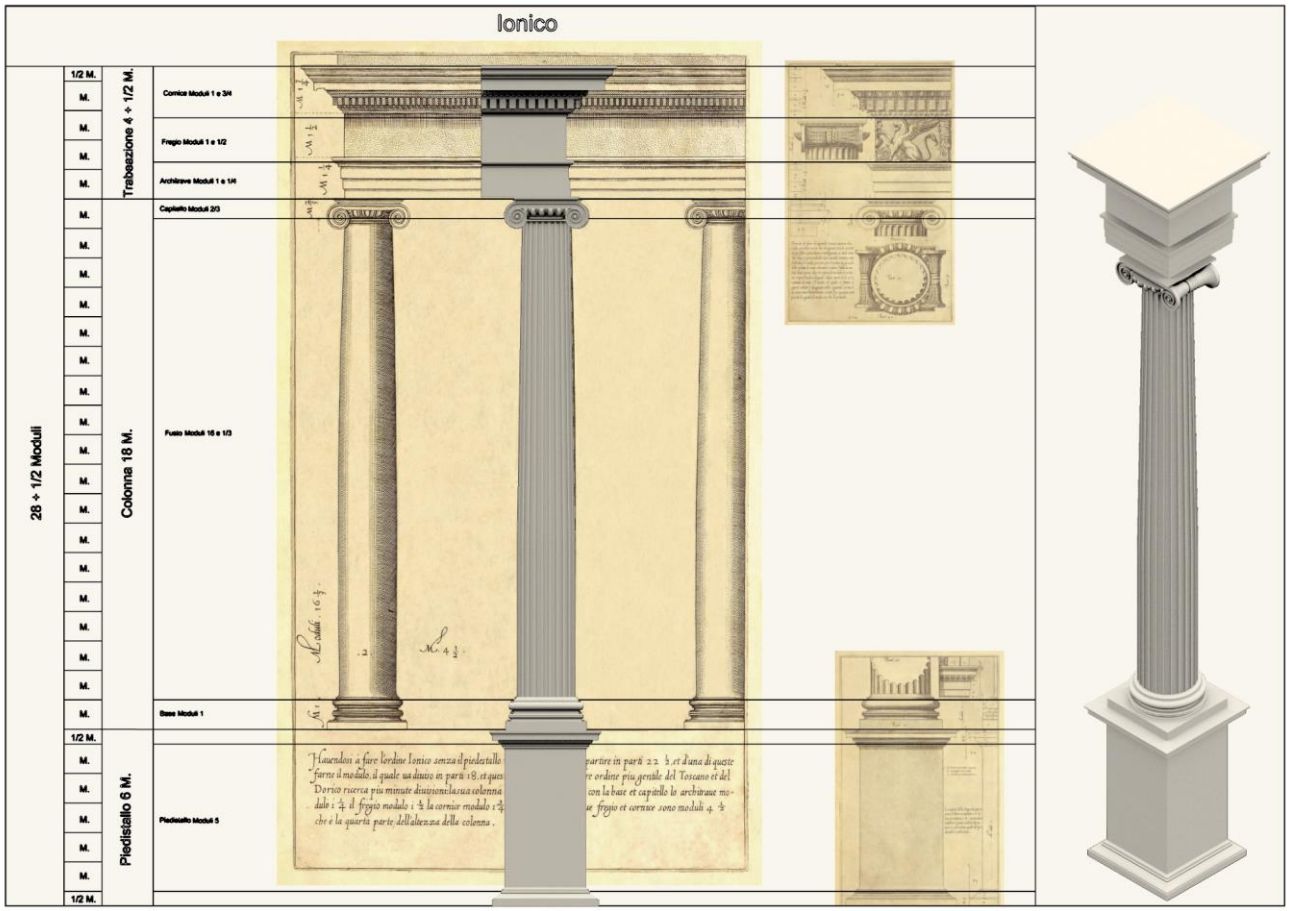

Fig. 12: Front and axonometric view of the 3D model of the Ionic order.

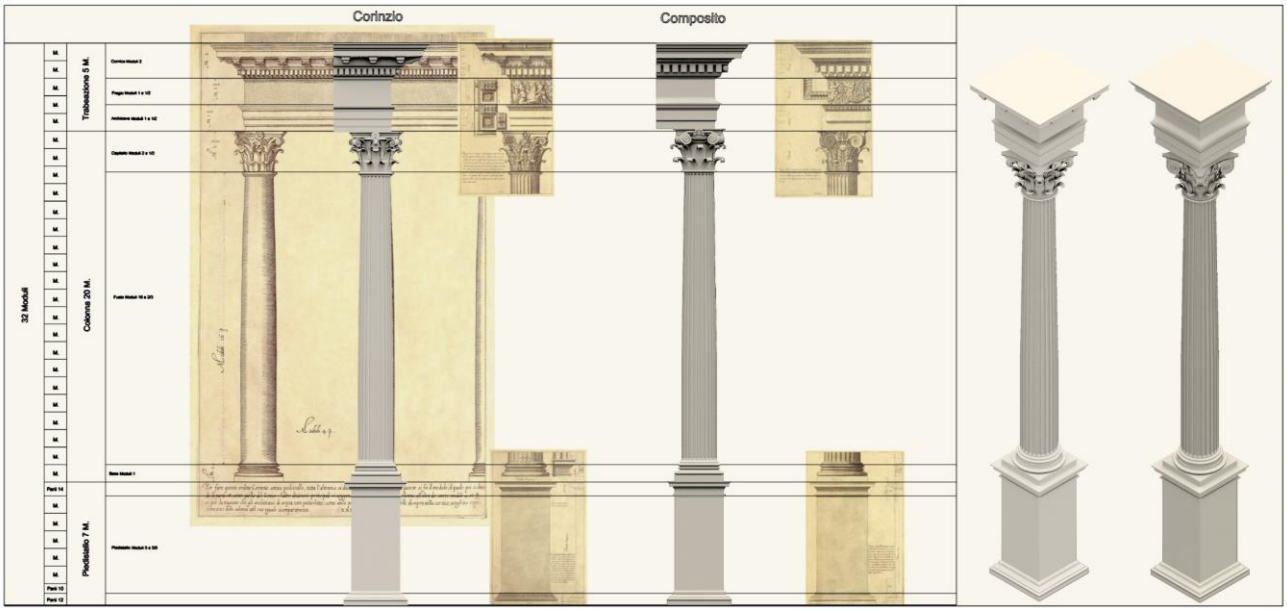

Fig. 11: Front and axonometric view of the 3D models of the Corinthian and Composite orders.

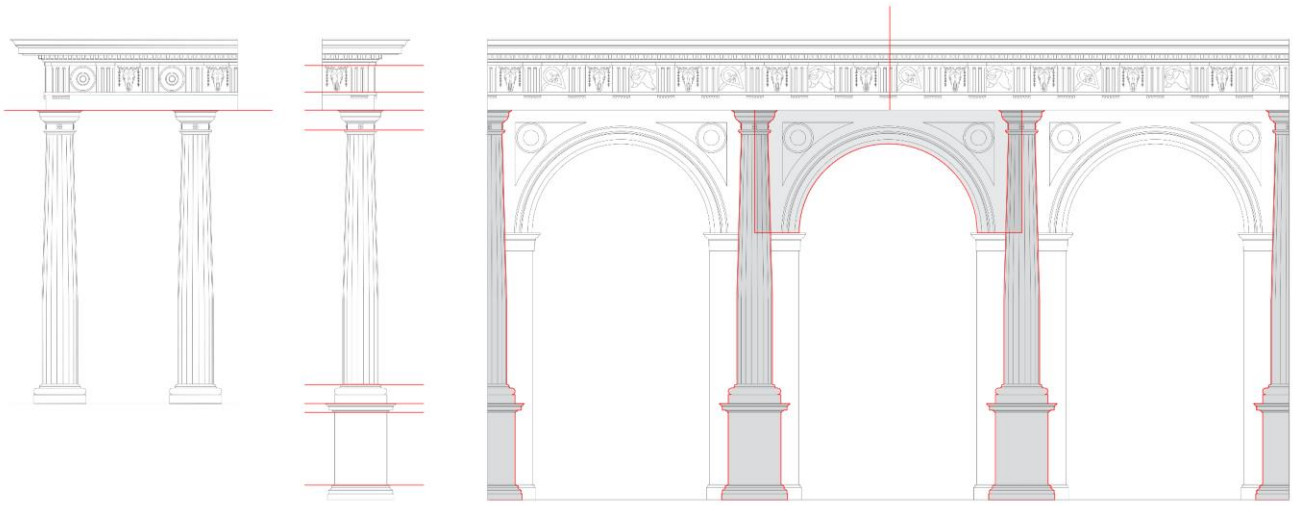

Fig. 13: Detail table regarding the decomposition of the models.

The infill percentage was chosen to ensure the solidity of the individual pieces, considering that they would be assembled and disassembled multiple times; material information pertains to chamber temperature and extrusion temperature.

These parameters influence the print regarding the final resolution of the printed object, the accuracy of details, speed, and, consequently, printing times. (Reddy et al., 2021).

5.2 Decomposition of Models and Completed Models

To proceed with the decomposition of the models, the guidelines described in the tables of the treatise were followed, reproducing the three systems represented in the tables: the simple order (base, column, entablature), the trilithic system (two columns and the entablature), and the wall system with arch (columns interspersed with arches and topped by the entablature) (Figs. 14 - 15).

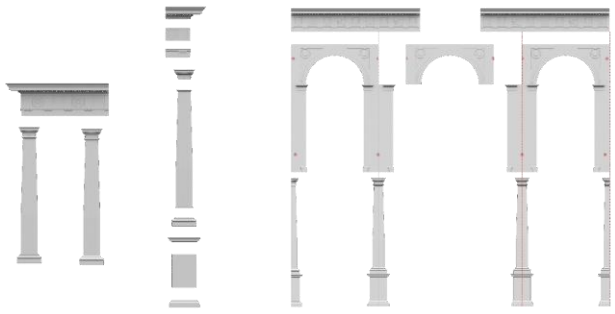

Fig. 14: Front view of the 3D models decomposed into pieces

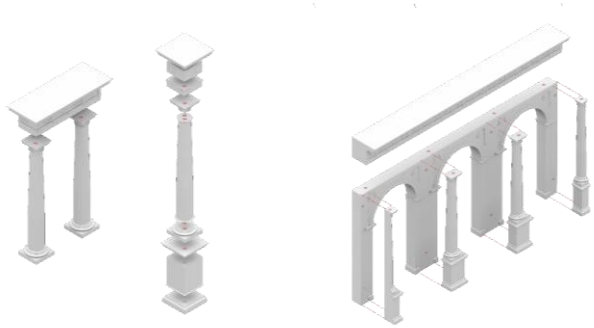

Fig. 15: Exploded axonometric view and subdivision of the models.

Regarding the decomposition of the models of the individual orders, although Vignola provides a detailed breakdown of the compositional elements (with the peculiar differences of each order), a simplified subdivision was followed. For the printed work, a classification of the elements based on a macro nomenclature common to all orders was chosen, namely: base of the pedestal, shaft of the pedestal, cap of the pedestal, base of the column, shaft of the column, capital, architrave, frieze, and cornice, respecting the subdivision provided by Vignola.

The final study model created is the system with arch, which consists of an arch situated between two columns and topped by an entablature. In the tables drawn by Vignola, this is represented as a continuous system; therefore, in its realization, a model with three arches and four columns was produced, sectioning the model just

after the outer columns. This system was designed and printed for only four orders, excluding the Composite order, as it is identical to the Corinthian order, thus following the representation provided in Vignola's treatise (Figs. 16, 17).

Fig. 16: 3D printed and disassembled models.

Fig. 17: 3D printed and reassembled models.

In each element, holes are designed during the modeling phase where magnets will be inserted. For elements to be stacked along a vertical axis, it was decided to place the magnets diagonally: an initial design with a single axial magnet allowed for the rotation of the various elements, enabling incorrect assembly; however, the insertion of two magnets ensures correct stacking. Once the model is printed, the magnets are fixed within the holes and positioned with attention to the two poles, ensuring that each element only connects in the correct position for easier reconstruction of the model.

Subsequently, wooden supports were designed and constructed, onto which Vignola's table was printed. Their purpose is to provide stability to the built model through pins that connect the model to the wood and allow for the visualization of the modular system and nomenclature, thanks to the printed material placed above.

The level of detail achieved did not necessitate any filling or sanding, an aspect that was considered from the early stages of design, as these are working models rather than display pieces, meaning they will be handled multiple times, and any filling would inevitably expose them to wear and damage. (Figs. 18, 19). (Paragraph 5 was written by Daniela Butera).

6. Conclusions

The result of what has been previously described aims to provide students with a teaching tool that complements traditional study methods. This is expected to occur through the tactile manipulation of architectural elements, which would promote active learning and improve the understanding of syntactical and compositional rules. In the future, there are plans to integrate the3D printed models witj augmented reality, which could facilitate the understanding of architectural terminology by adding interactive digital information to physical models.

Additionally, a "serious game" is planned to be structured into three levels of difficulty: a quiz for middle school students, where they would perform associations and recognitions, a simplified redesign of the reassembled models for art high school students, and an advanced test for architecture students, which would include a graphic exercise and the recognition of compositional elements of architectural orders.

Finally, it is planned to collect and analyze data derived from classroom observations during the implementation phase, analyzing learning outcomes related to the acquisition of knowledge, motivation and drawing skills (Ethier et al. 2022).

Fig. 18: Detail of the Doric order 3D printed model.

Figs. 19-20: Final models reassembled and placed in their respective wooden supports.

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